Introduction: This application note discusses various techniques for the measurement of phase noise using frequency domain analysis. Typical applications areas are characterization of phase noise in crystal oscillators, phase noise of synthesized sources and phase noise in PLL or other measurement systems. These methods should give good estimates of phase noise for frequencies upto 2 GHz. Frequency stability in the microwave region is not covered.

Concepts of frequency stability:

Instantaneous frequency can be defined as the time rate of change of phase, i.e.

$$2\pi\nu(t) \equiv \frac{d\Phi(t)}{dt} \equiv \dot{\Phi}(t)$$

where $\Phi(t)$ is the instantaneous phase of the oscillator, and the angular frequency is

$\omega \equiv 2\pi f$.

The instantaneous output voltage of a DUT may be written as

$$V(t) = \left[V_o + \varepsilon(t)\right]\sin\left[2\pi\nu_o t + \varphi(t)\right]$$

where V_o and ν_o are the nominal amplitude and frequency of the output, and it is assumed that

$$\left|\frac{\varepsilon(t)}{V_o}\right| << 1 \text{ and}$$

$$\left|\frac{\dot{\varphi}(t)}{2\pi\nu_o t}\right| << 1$$

for substantially all time t. Then,

$\Phi(t) = 2\pi\nu_o t + \varphi(t)$ and

$$\nu(t) = \nu_o + \frac{1}{2\pi}\dot{\varphi}(t)$$

Spectral Density of Phase Fluctuations S_φ(f)

This is the most basic measure of phase noise. It is the square of the Fourier transform of the time varying phase $\varphi(t)$ normalized to a 1 Hz bandwidth. The spectrum is normalized because when measuring a broadband signal such as noise, the amplitude of the spectrum changes with the bandwidth over which the measurement is made. Normalizing allows measurements with different bandwidths to be compared. If the noise is Gaussian in nature, the amount of noise in other bandwidths may be approximated by scaling the spectral density by the bandwidth. Using the above example of phase modulation, $S_\varphi(f)$ can be written as

$$S_\varphi(f) = \frac{\Delta\varphi^2{}_{rms}(f)}{B}$$

where $\Delta\phi_{rms} = \Delta\phi / \sqrt{2}$ and B is the bandwidth (in Hz) used to measure $\Delta\varphi_{rms}$. $S_\varphi(f)$ describes the phase noise at an offset f on both sides of the carrier.

Single Sideband Phase Noise L(f)

This is the most common way to express a phase noise spectrum. L(f) is defined as the ratio of power in a single 1 Hz sideband at an offset frequency f to the total signal power. Typically, when measuring phase noise, almost all of a signal's power is in the carrier, so carrier power can be substituted for total power, giving

$$L(f) = \frac{\text{Power in a single 1Hz sideband}}{\text{Carrier power}}$$

The units of L(f) are dBc/Hz (decibels relative to carrier per Hz bandwidth). One can see that this quantity can be easily measured using an RF spectrum analyzer(provided it has enough dynamic range).

In the case of small total phase modulation(total $\Delta\varphi_{rms}$<<1 radian), which is the norm when making phase noise measurements, one can relate L(f) to $S_\varphi(f)$ using phase modulation theory.

$$L(f) = 10\log\left[\frac{S_\varphi(f)}{2}\right] = 10\log\left[\frac{\Delta\varphi^2{}_{rms}}{2B}\right]$$

L(f) describes the phase noise at an offset f on one side of the carrier.

Spectral Density of Frequency Fluctuations $S_v(f)$

Phase and frequency fluctuations are directly related. $S_v(f)$ is the square of the Fourier transform of the frequency fluctuations $\Delta v(t)$ normalized to a 1 Hz measurement bandwidth.

$$S_v(f) \equiv \frac{v^2_{rms}}{BW} \quad [\mathrm{Hz}^2/\mathrm{Hz}]$$

This measure is useful for characterizing noise on FM signals.

Spectral Density of Fractional Frequency Fluctuations $S_y(f)$

To compare the frequency noise of sources with different carrier frequencies, one measures fractional frequency fluctuations $y(t) = \Delta v(t)/v_o$. $S_y(f)$ is defined as the square of the Fourier transform of the fractional frequency fluctuations y(t) referenced to a 1 Hz measurement bandwidth.

$$S_\varphi(f) \equiv \frac{\Delta y^2_{rms}(f)}{B} = \frac{S_v(f)}{v_o^2}\left[\frac{1}{Hz}\right]$$

Residual FM

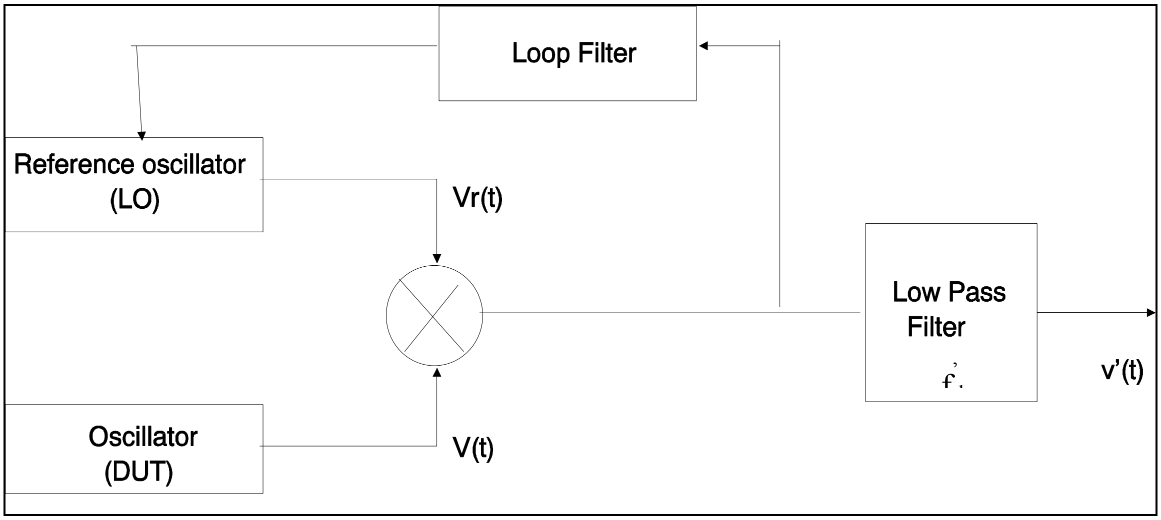
Residual FM is the rms of the frequency integrated over a certain band

$$\text{Residual FM} \equiv \sqrt{\frac{1}{b-a}\int_a^b S_v(f)df}$$

where a is the lower frequency and b is the upper frequency of the band of interest. Residual FM is commonly used to specify frequency stability in communication systems. Standard bands used are 50Hz - 3kHz, 300Hz - 3kHz and 20Hz - 15kHz. Due to averaging over frequency, residual FM does not convey information on the frequency dependence of the noise in the band.

Measurement Techniques for Frequency Stability

Heterodyning Techniques



Phase noise measurements are done by mixing the device under test with a very low phase noise reference oscillator which is "loosely" phase locked at 90^o to the device under test. Let the "ideal" reference be

$$V_r(t) = V_{Or} \sin 2\pi v_o t$$

Let the DUT's output signal be

$$V(t) = [V_O + \varepsilon(t)] \sin[2\pi v_{ot} + \varphi(t)]$$

The output of the mixer is then

$$\gamma V(t).V_r(t) = \gamma V_{Or}(V_O + \varepsilon)$$
$$[\sin 2\pi v_o t][\sin(2\pi v_o t + \varphi)]$$

$$= v(t) = \gamma \frac{(V_{Or}V_O)}{2`}\left(1 + \frac{\varepsilon}{V_O}\right)$$
$$[\cos\varphi - \cos(4\pi v_o t + \varphi)]$$

Assume that cos[φ(t)] has essentially no power in Fourier frequencies f in the region $f \geq f'_h$. The effect of the low-pass filter is to remove the second term on the right, i.e.

$$v'(t) = \gamma \frac{(V_{Or}V_O)}{2`}\left(1 + \frac{\varepsilon}{V_O}\right)\cos\varphi(t)$$

This separation of terms by the filter is correct only if $\left|\frac{\dot{\varphi}(t)}{2\pi v_o}\right| << 1$ for all t.

The relative phase of the oscillators is adjusted to be in approximate quadrature, i.e.

$$\dot{\varphi}(t) = \varphi(t) + \frac{\pi}{2}$$

where $|\varphi'(t)| << 1$. Now, we can use the following approximation $\cos\varphi(t) = \sin\varphi'(t) \approx \varphi'(t)$ and

$$v'(t) = \frac{\gamma}{2}V_{Or}V_O\varphi'(t) + \frac{\gamma}{2}V_{Or}\varphi'(t)\varepsilon(t)$$

If $\left|\frac{\varepsilon(t)}{V_O}\right| << 1$ for all t, then

$$v'(t) = \frac{\gamma}{2} V_{Or} V_O \varphi'(t) \text{ i.e.}$$

$v'(t)$ is proportional to the phase fluctuations. For different phase values, mixtures of amplitude and phase noise are observed.

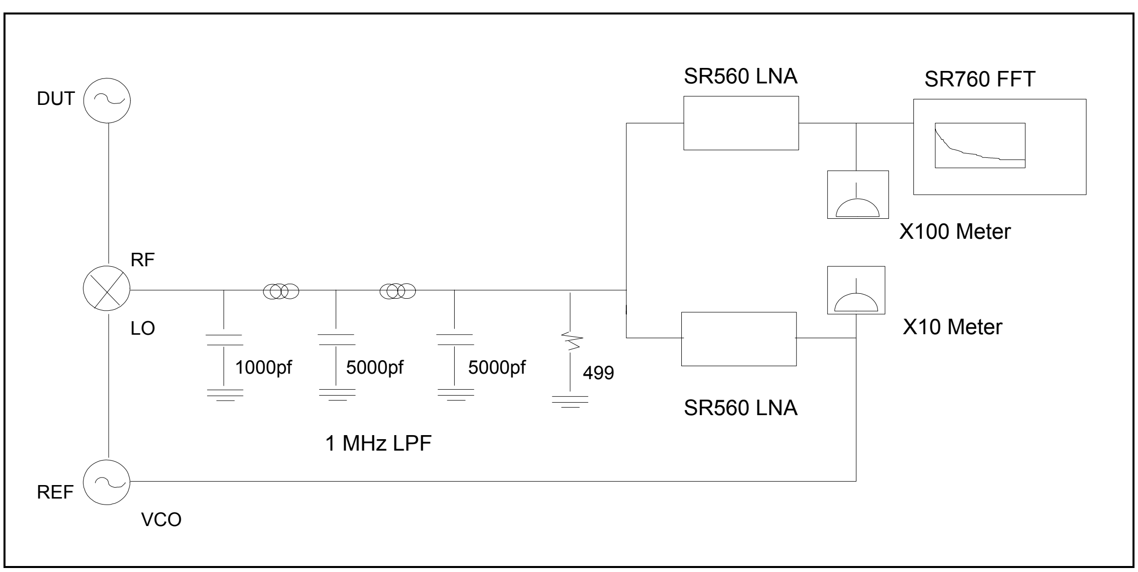
A VCO as a reference and a low noise amplifier as the feedback element serve as the phase locked loop to maintain quadrature. The output is analyzed by an FFT spectrum analyzer, like the SR760 or SR770, which can display the power spectral density(PSD) of this noise signal, normalized to a 1 Hz bandwidth.

A practical system for setting up such a measurement is shown below. It consists of the DUT which is a10MHz oscillator, a reference oscillator REF, and a mixer from Mini Circuits, model ZAD-3SH. The mixer output is low pass filtered by a 1MHz filter which essentially filters out the high frequency component of 2w.

The output is then fed through an SR560 Low Noise Voltage Amplifier, with gain set to 100, 300kHz bandwidth and a 6 dB/octave slope for the built in low pass filter. An SR760 FFT Analyzer is used to characterize the spectral density of the output signal. A feedback is provided by another SR560 with the gain set to 10, and the low pass filter set to a 1 Hz bandwidth at 6 dB/octave. Two analog meters are used to tune the VCO, one for course tuning and one for fine tuning. From the measurement of power spectral density ($V_{rms}/\sqrt{Hz}$), the single sideband phase noise is given by

$$\frac{dBc}{Hz} = 20\log\left[\frac{Vn(rms/\sqrt{Hz}}{K_\varphi}\right] - 3$$

The 3 dB correction accounts for the fact that when mixed to zero beat note, we measure the combined power of the upper and the lower sidebands. Noise of the reference should also be corrected for. If the reference is much quieter than the DUT, no correction is required. The only other item which must be measured is K_φ, the phase detector sensitivity.



This must be measured for each device tested, as it will depend on the matching of the DUT into the mixer, the output level, the output impedance of the DUT, the LO drive level etc.

To measure K_φ(V/radian), disconnect the feedback, replace the SR760 with an SR620 Time Interval Counter, and characterize the beat note between DUT and the LO. Measure the period (T) and the rise time (t_r) to rise between thresholds separated by ΔV. Then

$$K_\varphi = \frac{\Delta V \bullet T}{2\pi t_r}$$

This measurement can be easily made with the SR620. For example, if the SR620 thresholds are set to +/- 2.50 V and T=1.10s and t_r=0.0063s then

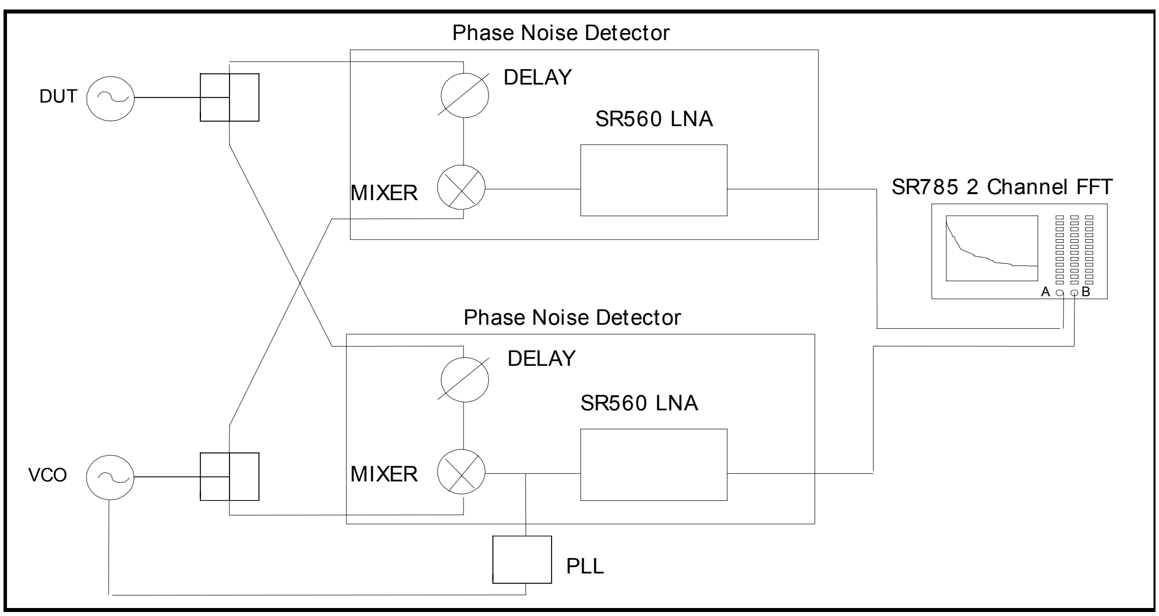
$$K_\varphi = \frac{5 \times 1.01}{2\pi \times 0.0063} = 127V / rad$$

The SR620 should be set to DC coupled and high input impedance.

Cross Correlation Phase Noise Method

The noise floor of a typical phase noise measurement system is determined by the phase detector and the amplifiers that follow. If we measure phase noise using two such test systems, then the output noise is uncorrelated except for the component due to the phase noise between the oscillators. This technique improves the noise floor of measurement, but takes longer since averaging is needed to remove the uncorrelated noise. A major component of the system is a 2 channel FFT cross correlation analyzer (like the SR785), which has wide real time bandwidth and a fast processor for upto 512 averages/s.

A typical setup is shown in the figure below. The signals from the LO and the DUT are split and connected to two double balanced mixers.



The output of each mixer is low pass filtered and amplified. The signals are then fed into the cross-correlation FFT analyzer. The random noise of the test setup is uncorrelated while the correlated noise is the actual noise of the input signals along with some leakage due to the power splitters.

To calibrate the system, K_d is measured as before. The feedback is broken and the beat signal's period T and risetime t_r are measured using an SR620 Time Interval Counter.Then

$$K_d = \frac{\Delta V \bullet T}{2\pi t_r}$$

The signals at the mixers are maintained in quadrature as before by a phase lock loop or a delay line.

The spectral density of the cross-spectrum now gives the phase noise of the DUT(assuming negligible contribution from the LO).

References:

1. Byron E. Blair, *Time and Frequency: Theory and Fundamentals, Chapter 8, NBS* (1973)
2. Warren F. Walls, *Cross-Correlation Phase Noise Measurements*, Femtosecond Systems
3. F.L. Walls, A.J.D. Clements, C.M. Felton, M.A. Lombardi and M.D. Vanek, *Extending the range and accuracy of phase noise measurements*(1988)