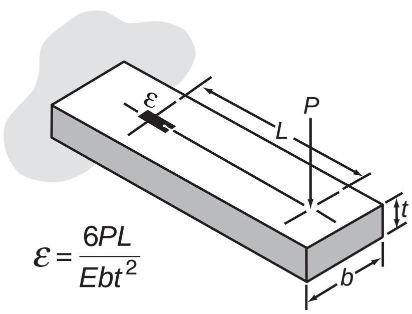
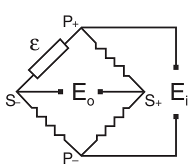
Force and Torque Measurement

When evaluating a new design, the stress analyst will usually measure not only the mechanical strains in the test structures, but the forces and torques that produce the strains as well. This is especially important when the objective is to determine the response of the design to known or anticipated loading conditions with an eye toward reducing weight and establishing safety margins. For measuring these non-strain loading parameters, the experienced analyst will generally opt to utilize commercially available transducers specifically designed for that purpose. But when the use of a ready-made transducer is impractical - or even impossible - strain gages can often be installed on some component of the test structure or loading fixture in a configuration that will enable the applied forces and torques to be measured directly. The physical arrangements and electrical circuits necessary for achieving these measurements are described below.

Bending Beam - Quarter Bridge
(Not recommended)



$$\varepsilon = \frac{6PL}{Ebt^2}$$

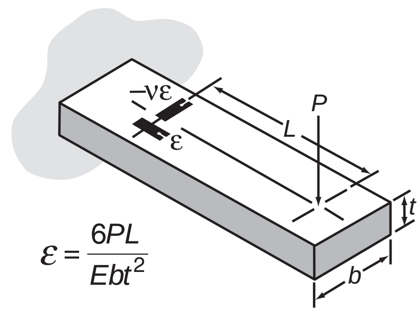


$$\frac{E_o}{E_i} = \frac{F\varepsilon}{4+2F\varepsilon}$$

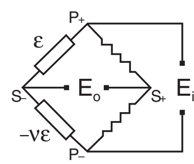
This single longitudinal gage configuration will respond to bending loads but is unaffected by torsional loads if the gage is mounted on the centerline. Care must be taken with how the load is applied, because transducers utilizing this configuration will also respond to any axial loads that may be present. Since this configuration produces a small amount of nonlinearity (approximately 0.1% for each 1000 microstrain) and is sensitive to changes in temperature, the following " half-bridge " configuration is generally preferred. When a half bridge cannot be used, the sensitivity to temperature for a single active gage configuration can be minimized by using the proper self-temperature-compensated strain gage and by zero-balancing before the load is applied.

The use of a three-leadwire circuit is recommended for all quarter-bridge installations used to make static measurements

Bending Beam -Half Poisson Bridge



$$\varepsilon = \frac{6PL}{Ebt^2}$$

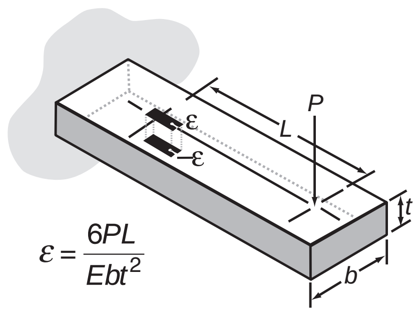


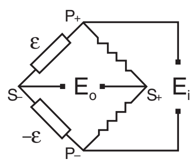
$$\frac{E_0}{E_i} = \frac{F\varepsilon(1+\nu)}{4+2F\varepsilon(1-\nu)}$$

where F = Gage factor
ε = Strain
ν = Poisson's ration

Because the longitudinal gage and the transverse "Poisson" gage are in adjacent arms, the resistance changes of thermal origins will be cancelled in this version when both active gages and the specimen experience like changes in temperature. The bridge output is increased by a factor of approximately $(1 + \nu)$ and the nonlinearity is reduced to approximately $[(1 - \nu/10)]$ % per each 1000 microstrain of longitudinal strain.

Bending Beam -Half Bridge

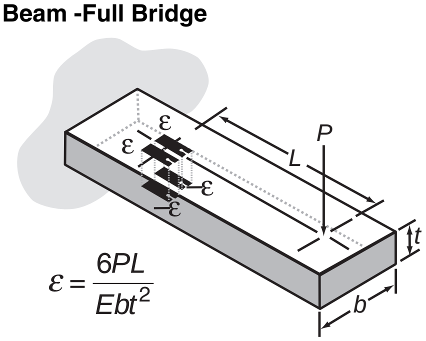


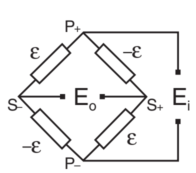


$$\frac{E_0}{E_i} = \frac{F\varepsilon}{2}$$

In this configuration, two axial gages are used. The gage on the lower surface is located precisely under the gage on the top surface and they measure bending strains that are of equal magnitudes but of opposite signs. Any resistance changes in the active gages resulting from strains of like sign produced by axial loads will be cancelled because the two active gages are in adjacent arms of the Wheatstone bridge. Likewise, resistance changes of thermal origins will be negated when both gages and the specimen experience the same changes in temperature. And because the strains produced by bending loads are of equal magnitude but opposite sign, the bridge output is not only linear but is nominally double that produced by a single active gage under the same conditions.

Bending Beam -Full Bridge

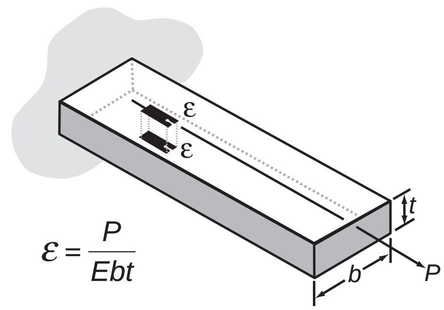


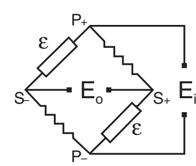


$$\frac{E_0}{E_i} = F\varepsilon$$

This four-gage version is the most popular bending beam configuration. The linear bridge output is twice that of the preceding half-bridge version. Note that the two gages on the top surface are in opposite arms of the Wheatstone bridge, as are the two gages on the bottom surface.

AxialColumn - 2 Gages in Opposite Arms

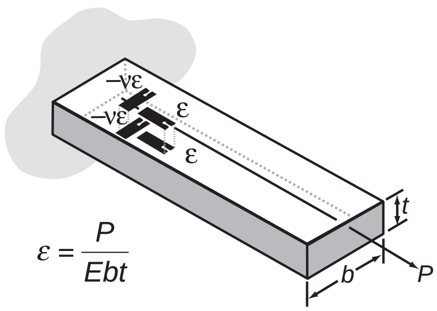




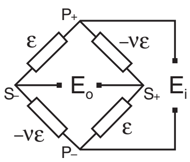
$$\frac{E_o}{E_i} = \frac{F\varepsilon}{2 + F\varepsilon}$$

The physical configuration of the gages is the same as that used for the bending half bridge . But because the two active gages are now electrically connected in opposite arms of the Wheatstone bridge, this configuration cancels bending strains with equal magnitudes and opposite signs. The magnitude of the bridge output resulting from axial loads is relatively high (because of the additive effect) but is nonlinear (approximately 0.1% per each 1000 microstrain produced in the column by axial loads). And because any thermal output from the active gages is additive for this configuration, temperature compensation is the poorest of any configuration shown here.

Axial Column - Full Poisson Bridge



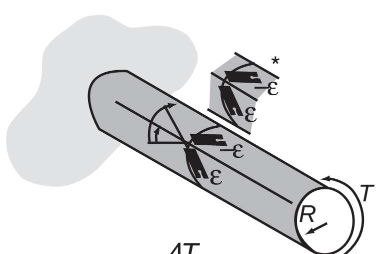
$$\varepsilon = \frac{P}{Ebt}$$



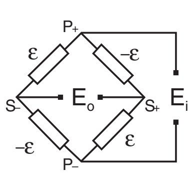
$$\frac{E_o}{E_i} = \frac{F\varepsilon(1+\nu)}{2+F\varepsilon(1-\nu)}$$

This "full-bridge" configuration with a longitudinal gage and transverse "Poisson" gage on both top and bottom surfaces is the most popular for axial loads. The output is not only higher by approximately a factor of $(1 + \nu)$ than for the previous two-gage version but, is also less nonlinear (approximately $[(1 - \nu)/10]$ % per 1000 microstrain produced by axial loads). This version has good temperature compensation because gages are present in all adjacent arms of the bridge. Note that both gages on a given surface are in adjacent arms of the bridge.

Torque - Full TorsionBridge



$$\gamma_{max} = 2\varepsilon = \frac{4T}{\pi E R^3}(1+\nu)$$



$$\frac{E_0}{E_i} = \frac{F\gamma_{max}}{2}$$

Like the full-bridge configuration for bending loads, this torsional version has a linear output and good temperature compensation. All effects of both bending and axial loads are cancelled in this most popular design for torque measurement. However, very accurate gage orientation and placement of all four gages is crucial for success.

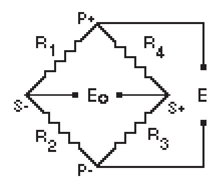
Fundamental Considerations

High-quality transducers utilizing strain gages as the primary sensing element incorporate sophisticated techniques to minimize thermal effects, nonlinearities, hysteresis, and other sources of error. These same techniques can be engaged when a portion of a structure or loading frame must be adapted to function as the spring element. For now, however, considerations will be limited to how the unique characteristics of the Wheatstone bridge itself can be utilized for measuring force and torque. And only the most significant unresolved errors or potential errors associated with the various configurations will be noted.

The output from the Wheatstone bridge can be expressed as:

$$\frac{E_0}{E_i} = \frac{R_1}{R_1+R_2} - \frac{R_4}{R_3+R_4} \qquad (1)$$

where E_0 is the bridge output voltage, E_i is the bridge excitation voltage, and R_1, R_2, R_3, and R_4 are the resistances of the gages in the bridge.



Note that changes of resistance in adjacent gages (R_1 and R_4, for example) have the same (or numerically additive) effect on bridge output when the changes are of opposite sign. When the changes in adjacent arms are of the same sign, they have opposite (or numerically subtractive) effects. Conversely, the effects of resistance changes in opposite arms (R_1 and R_3, for example) are reversed: changes in resistance with like signs in opposite arms have the same effects on the output and changes with unlike signs in opposite arms have opposite effects. These phenomena are used later to eliminate the effects of bending strains on the measurement of axial strains, the effects of axial strains on measurements of bending strains, eradication of thermally induced apparent strain, and even elimination of nonlinearity in the output from unbalanced Wheatstone bridges.

The bridge is balanced and produces no output when:

$$\frac{R_1}{R_1+R_2} = \frac{R_4}{R_3+R_4}$$

or when:

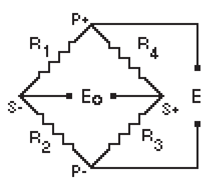
$$\frac{R_2}{R_1+R_2}=\frac{R_3}{R_3+R_4}$$

A change in resistance of one or more of the gages will unbalance the bridge so that an output signal is produced. Equation (1) can be used to determine the output of any unbalanced bridge simply by adding the resistance change of each affected resistor to the initial value of that resistor.

Sample Calculation

As a sample calculation of the output from a bridge, assume that R_1 is an active strain gage that has undergone a change in resistance, ΔR_1, when the test specimen to which it is bonded has been subjected to stress. Equation (1) can then be rewritten as:

$$\frac{E_0}{E_i}=\frac{R_1+\Delta R_1}{R_1+\Delta R_1+R_2}-\frac{R_4}{R_3+R_4} \qquad (2)$$



Assuming that all gages have the same initial resistance, then equation (2) reduces to:

$$\frac{E_0}{E_i}=\left(\frac{R_1+\Delta R_1}{2R_1+\Delta R_1}-\frac{1}{2}\right)=\frac{\Delta R_1/R_1}{4+2\left(\Delta R_1/R_1\right)} \qquad (3)$$

Noting that the relative resistance change of a strain gage ($\Delta R/R$) is equal to the product of the gage factor (F) and the strain producing the resistance change (ε), the bridge output for our single active gage example can then be expressed in terms of strain:

$$\frac{E_o}{E_i}=\frac{F\varepsilon}{4+2F\varepsilon} \qquad (4)$$

While usually small in comparison to the number 4, the 2 $F\varepsilon$ term in the denominator gives rise to some nonlinearity for this configuration. The magnitude of this error is approximately 0.1% per each 1000 microstrain for bridges with single active gages. In most stress analysis applications this very small measurement error can be ignored, especially at low strain levels.

The preceding sample calculation yields the output from a quarter-bridge circuit measuring strains produced by bending a beam. The same can also be used with equal success for any number of other active gage configurations for measuring force and torque with Wheatstone bridge circuits.

The design of good transducers is a highly complex and difficult task that is not to be undertaken lightly. Whenever possible, the stress analyst is well advised to purchase a commercially available unit from "off-the-shelf". The preceding gage configurations are applicable only to the most elementary of spring elements. When the necessity arises for constructing a "home-grown" transducer based on them, then do not hesitate to do so. Just be aware of their unique capabilities and any inherent limitations they may pose.