We have

$$Z_5 = \frac{Z_3 Z_4}{Z_3 + Z_4 + Z_p + Z_q} = \frac{Z_3 Z_4}{Z_l}$$

$$Z_4' = Z_4 \cdot \frac{Z_p + Z_q}{Z_l}$$

$$Z_3' = Z_3 \cdot \frac{Z_p + Z_q}{Z_l}$$

and

$$Y_5 = \frac{Z_5 \cdot Y_A Y_B}{1 + Z_5(Y_A + Y_B)} \approx \frac{Z_3 Z_4 Y_A Y_B}{Z_l}$$

There are thus two main effects on the measured admittance by connecting terminals 3 and 4. Z_m has been reduced by an amount $(Z_3+Z_4)^2/Z_l$ due to the shunting of Z_3+Z_4 with Z_p+Z_q, and an additional admittance Y_5 has been added directly to Y.

Typical values are $Z_3+Z_4=10$ mΩ and $Z_l>1$Ω so the reduction in Z_m is <0.1 mΩ which is about the limit of definition with ordinary connectors. Y_5 is very small and needs to be considered only when measuring very small admittances. It can usually be reduced to a negligible value. Since only the sum of Z_p and Z_q appears in the circuit, only one choke need be used and it is also more effective to have all the turns on one core.

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The Precise Measurement of Current Ratios

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***Abstract*—The development of the current comparator, a three-winding current ratio transformer, is reviewed and its characteristics as an alternating current ratio standard are analyzed. Particular attention is given to the use of magnetic shielding and its effect on the accuracy and usefulness of the device. Some error characteristics of three types of audio-frequency current comparators are given and possible applications discussed. These include the calibration of current transformers and impedance comparisons.**

The adaptation of this device to dc operation is made possible by modulation techniques. A 20 000-ampere self-balancing direct current comparator, designed specifically for the calibration *in situ* of transductors or direct current transformers, is described. The application of this comparator to the calibration of shunts at high currents also is discussed and some results presented.

INTRODUCTION

THE CALIBRATION of current transformers requires the determination of the ratio of two currents. Over the last several years the accuracy requirement of this determination has increased steadily. In addition the operating frequency has increased from the usual power frequencies to the limits of the audio-frequency range. To meet this demand the National Research Council of Canada (NRC) decided, about five years ago, to review their calibration procedures with the aim of establishing the ratio stability of high quality current transformers and of developing calibration techniques capable of accuracies at least as good as that stability.

Several decades ago, the actual ratio of a current transformer was significantly different from the turns ratio of its windings. In order to achieve a given nominal ratio with a minimum of error, very ingenious means were invented to realize a fraction of a turn. The fact that this could not be done in a straightforward manner was then thought to be a serious disadvantage. With modern materials and design techniques, however, the actual ratio of a transformer is very nearly equal to its turns ratio and the integral nature of a turn because its most important characteristic. With the use of tape-wound cores of high permeability material it has been possible to build multiratio precision current transformers whose ratio error at power frequencies is less than 100 parts per million (ppm) both in magnitude and in phase.

Two design techniques appear to be important if low errors and good ratio stability are to be achieved. The

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first is mechanical protection of the high permeability core which is very strain sensitive. The second is the use of sector windings which reduce considerably the effect on ratio of ambient and stray magnetic fields. In this type of winding the secondary consists of several parallel windings, each having the same number of turns and occupying equal sectors on the toroidal core.

Properly-designed modern precision current transformers have an error characteristic which is relatively unaffected by temperature, humidity and stray magnetic fields. Their ratio, however, remains rather sensitive to magnetization of the core which can be caused by switching transients. When carefully demagnetized their ratio stability is about one or two parts per million.

The well-known bridge methods of calibration [1], [2] which use standard resistances, capacitances, and mutual inductances do not offer much promise of being able to achieve accuracies of this order of magnitude, especially where large ratios and currents in the 100's or 1000's of amperes are involved. The comparison of two current transformers of the same ratio is capable of being carried out to this order of resolution but for many years, the accuracy of any calibration based on such a comparison was limited by the calibration of the current transformer used as a standard. This limitation was removed by Forger [3] when he developed build-up or boot-strap calibration methods for standard current transformers.

For the calibration of current transformers to the highest precision the use of another current transformer as a current-ratio standard is not without limitations. As long as the errors of the standard are negligibly small no difficulties arise and the calibration equipment can be made direct reading. When, however, the transformer tested is of the same or higher quality than the standard transformer the results have to be corrected for the error of the standard. This correction is rather tedious because of the nonlinear characteristic of the standard current transformer. In addition the ratio stability of the test transformer can no longer be investigated.

Ideally a ratio standard should have a lower error and better ratio stability than the device to be calibrated. In theory such a standard is not unrealizable provided it is based on an entirely different principle of operation. The search for such a device has led to the development of a ratio standard called the current comparator. The principles involved in the operation of this device are not new; it appears however that its full capabilities have never before been fully appreciated. The development of the current comparator for current-transformer calibration has presently been carried out for ratios up to 2000/5 A at the power frequencies and up to 500/5 A at audio frequencies as high as 16 kc/s. The current comparator has also been developed for the measurement of direct currents up to 20 000 A, at ratios as high as 20 000/1. This paper intends to review these developments.

As is often the case, when a new tool for precise measurement becomes available several other applications become apparent. These applications include impedance comparisons and the calibration of dc shunts at high currents.

THE CURRENT COMPARATOR

The current transformer is a ratio transformer of the two-winding type. The current comparator is a three-winding ratio transformer. In its simplest form [Fig. 1(a)] it consists of two ratio windings and a detection winding uniformly distributed on a toroidal core. When the currents in the two ratio windings which magnetize the core in opposite directions are such that zero voltage appears in the detection winding a current ratio is established which corresponds, to a high degree of accuracy, to ampere-turn balance in the two ratio windings. Deviation of this balance condition from exact ampere-turn balance constitutes the magnetic error of the current comparator. The current ratio corresponding to ampere-turn balance is not exactly equal to the turns ratio because of leakage currents caused mainly by capacitances across and between windings, and from windings to ground. The deviation of the current ratio from turns ratio at ampere-turn balance is called the capacitive error of the current comparator. The total error of a current comparator as a current ratio device is the sum of its magnetic and capacitive errors. At power frequencies the magnetic error is predominant while the capacitive error becomes more important at higher frequencies.

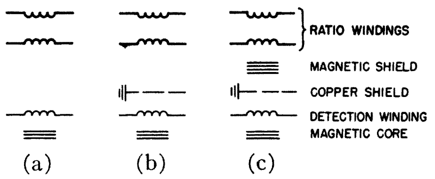


Fig. 1. Major components of a current comparator.

The Magnetic Error and Its Reduction

As an ampere-turn balance indicator the current comparator is essentially the physical embodiment of one of the fundamental laws of electromagnetism: the circuital law. This law states that the line integral of the magnetizing force H along a closed loop is unaffected by all currents in the vicinity of the closed loop except those passing through the loop.

$$\oint H dl = \sum I$$

In designing an ampere-turn balance indicator based on this law a detector is sought whose output is proportional to the integral $\oint H dl$. The voltage induced in a uniformly-distributed detection winding around a toroidal core of high permeability provides a detector which is very sensitive to currents linking the core and relatively insensitive to currents in the vicinity of the core but not linking it. It is however also sensitive to the potential differences between the detection winding and the current carrying windings. This can be eliminated by a copper shield which provides electric shielding [Fig. 1(b)]. The sensitivity of the detector to ambient magnetic fields can be further reduced by surrounding the detection winding with a toroidal magnetic shield [Fig. 1(c)].

The early current comparators built at NRC incorporated only a copper shield and no magnetic shield [4]. For low ratios, up to 12/1, the magnetic error of these current comparators could be kept below 1 ppm at the power frequencies by suitable design of the ratio windings. These windings were wound in a bifilar fashion using series-parallel connection to obtain ratios different from 1/1. Attempts to maintain this low error for larger ratios proved unsuccessful however; for practical reasons bifilar winding techniques could not be used for these larger ratios. The error increase with ratio is caused by the fact that the effect of a current carrying conductor linking the core depends not only on the magnitude of the current but also on the physical location of the conductor with respect to the core. With bifilar winding techniques the physical separation of the two-current carrying-conductors is minimum and good cancellation of the opposing effects of equal currents can be realized. For the larger ratios, this physical separation had to be increased for practical reasons and this led to an increase of the magnetic error. With suitable magnetic shielding these errors could be kept below 1 ppm at power frequencies for ratios as large as 2000/5 A even when using feed-through primary windings whose turns can be made to occupy any position around the core [5].

The reduction of the magnetic error by magnetic shielding can best be investigated with the "probe experiment" (Fig. 2). The probe consists of a long rectangular coil which is made sufficiently narrow to enable it to be inserted through the current-comparator core. When current is made to flow in this coil the detection-winding voltage should remain zero since no net ampere-turns are impressed upon the core. In practice, a voltage proportional to the probe current appears across the detection winding and it was found that this voltage, which represents the magnetic error, depends on the position of the probe. This voltage could be caused by the lack of uniformity of the detection winding. However a non-uniformity as high as ten percent would be required to explain the observed results. Another possible cause is nonuniformity of the permeability of the core along its circumference. In a tape-wound toroidal core a nonuniformity of this magnitude appears unlikely. It is however the only explanation of the observed fact we have been able to bring forward and the following reasoning may make this explanation less difficult to accept. At balance, the average flux density of the core is zero and the actual flux density in any part of the core is very low. At these very low flux densities the magnetic permeability of the material is not constant but increases with increasing flux density. The magnetic flux produced by the probe is symmetrical in the air portion of its circuit (Fig. 3). When this symmetrical flux penetrates the current-comparator core it should divide into two symmetrical portions which link, the detection winding in opposite directions if the permeability of the core is uniform. If however one of these components were only slightly larger than the other it would produce an increase of flux density in that half of the core and a corresponding decrease of flux density in the other part of the core. These changes in flux would generate changes in permeability which would increase the initial assymmetry of the flux distribution. It appears likely that a positive feedback situation exists which makes the uniform permeability condition an unstable one.

Two methods have been found to reduce the magnetic error:

1) The correction winding method.
2) The magnetic shield method.

In the first method the nonuniform characteristic of the core permeability is compensated for by a matching nonuniform winding distribution of the detection winding. For single frequency operation this technique is capable of a 10/1 reduction in magnetic error. The technique is cumbersome, however, and its effectiveness is greatly reduced as the frequency range of the current comparator is increased. For operation over the whole audiofrequency band the effect of the correction winding may even become deterimental at certain frequencies. No such limitations exist for the second method which consists in surrounding the toroidal detection winding and its copper shield with a hollow-toroidal magnetic shield. By this method the magnetic error can be reduced by a factor of 100/1 to 1000/1 over the whole audio-frequency range. In addition the use of a suitable magnetic shield makes the current-comparator balance condition practically independent of the magnetization of the measuring core. When this core is purposely magnetized with direct current the sensitivity of the device may drop considerably, thus giving a positive indication to the experimenter, but the balance condition is not measurably affected.

It should be remembered that while the electric shield and the magnetic shield are considered as entirely separate components, their action is not entirely separable, particularly at the higher frequencies. An electric shield made of copper is quite effective in reducing the magnetic error. Its effectiveness increases with increasing frequency and compensates the decreasing effectiveness of the magnetic shield. Another advantage of a copper shield is that it can be used between the two ratio windings where the magnetic-field intensity is high. The copper shield is not subject to saturation and retains its full efficiency. In contrast a magnetic shield cannot be used in this location because of saturation.

The physical size of the electromagnetic shield used in current-comparator construction depends to a great extent on the function it has to perform. In connection with closely-coupled ratio windings it can be used to

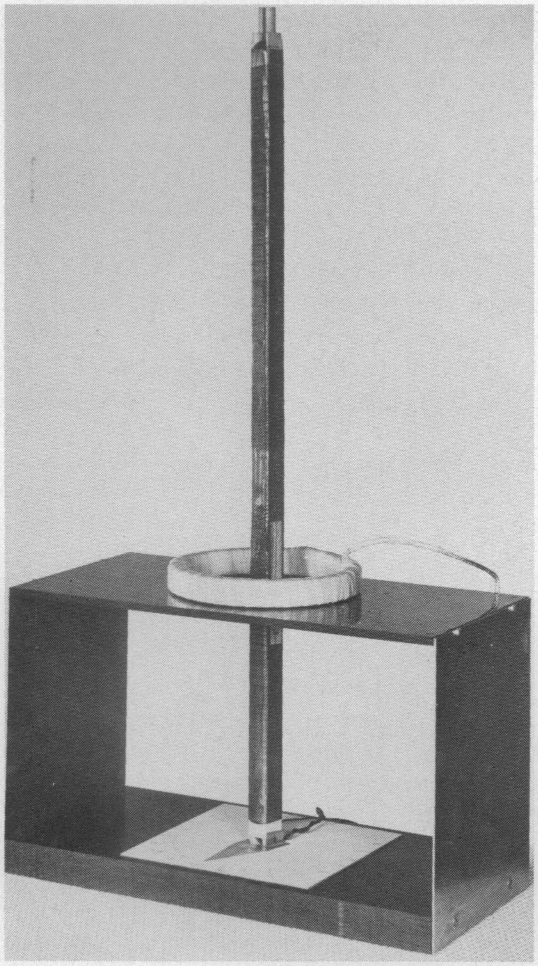


Fig. 2. The "probe" experiment.

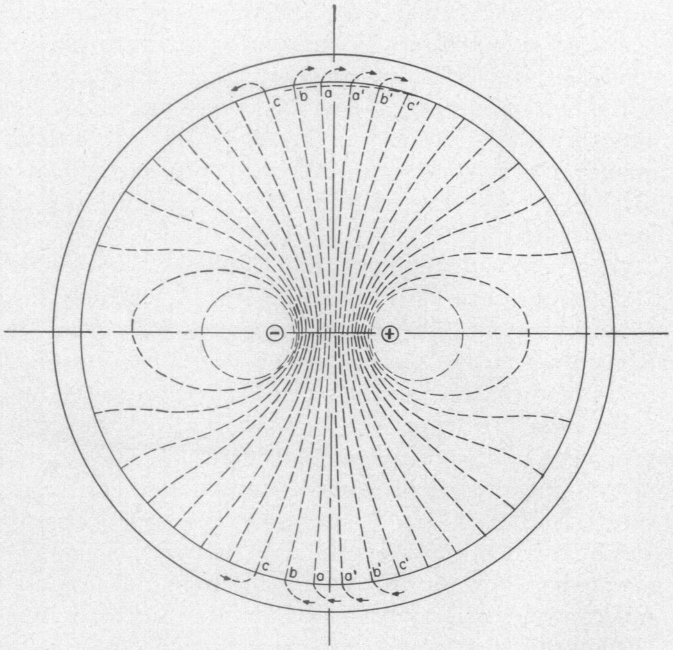


Fig. 3. Magnetic field around toroid.

reduce the small residual error even further. Since the leakage fluxes in this case are not very large only a small cross section shield is necessary. The advantages of magnetic shielding for this purpose were first recognized by Thompson [6]. When close-coupled ratio windings are not possible a larger cross section shield is required to keep the error low. A good example of this is the feed-through current comparator in which the primary turns are inserted through the core as required [5]. Particularly when primary currents of thousands of amperes are involved, the cross section of the magnetic shield required to keep the error below 1 ppm (at power frequencies) becomes considerably larger than the cross section of the error-sensing core. This is dictated by the requirement to keep the shield from local saturation. An even larger cross section of the shield is sometimes used for other than shielding purposes [7]. The magnetic shield can be made to perform other functions such as the control of energy transfer from one ratio winding to the other. Examples in which this energy-transfer function dictates the required cross section of the shield will be given later in the paper.

The Capacitive Error and Its Reduction

The current ratio corresponding to balance in a current comparator depends on where the currents are measured and on the potentials to ground of the ratio windings. It is convenient to define the current ratio of a current comparator as the ratio of the current leaving one terminal of the primary winding to the current entering the terminal of the same polarity of the secondary winding, when both these terminals are at ground potential. The terminals of same polarity where the currents are measured are called the marked or measuring terminals. The other terminals, the unmarked terminals will not be at ground potential because of the voltage drops in the windings. The potential differences existing within the current comparator are the origin of capacitive leakage currents which cause the ampere turns imposed on the core by the currents in any one ratio winding to be different from the product of the number of turns of that winding and the current at the marked terminal. This difference is called the ampere-turn error of the winding. The resulting capacitive error of the current comparator is the difference between the two ampere-turn errors of the ratio windings.

The capacitive error of a current comparator depends mainly on its ratio. At the 1/1 ratio, perfect winding symmetry leads to zero capacitive error. At other ratios, winding symmetry cannot be maintained and a capacitive error is unavoidable. While the potential differences and winding capacitances can be controlled individually by design, their effects are interrelated. To reduce the voltage drops in the ratio windings their leakage impedances have to be reduced. This can be accomplished by reducing the spacing between the windings but this in turn increases the winding-to-winding capacitance and the two effects tend to cancel each other. The winding-to-winding capacitance can however be reduced to zero by the use of an electrostatic shield between the two ratio windings. When this is done only the shunt capacitance and capacitance to ground of the ratio windings remain. The effects of these two capacitances on the ampere-turn error of a winding are of opposite sign and their magnitudes depend mainly on the number of turns of the winding.

When a magnetic shield is used a technique is available by which the variation of capacitive error with ratio in a multiratio comparator can be reversed so that the capacitive error becomes maximum at the 1/1 ratio, where it can be measured readily, and decreases as the ratio is increased. This technique is called "compensation." It consists in connecting the ratio winding with the largest number of turns in parallel with another winding which has the same number of turns but which is located inside the magnetic shield. This forms a composite ratio winding consisting of an outside ratio winding and an inside "compensation" winding (Fig. 4). At balance most of the current carried by this composite winding will flow through the outside winding. This is because the two outside ratio windings together with the magnetic shield form a current transformer. Only the error current of this current transformer will flow through the compensation winding. The voltage drop across the compensation winding is consequently small and, since the same voltage drop exists across the composite winding, it can be said that the ratio winding has been compensated for voltage drop.

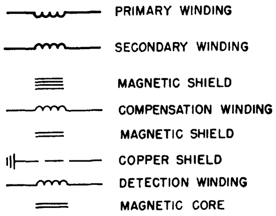


Fig. 4. Major components of a compensated-current comparator.

In the compensated current comparator the shield is excited and the voltage induced by this flux in the outside ratio winding very nearly cancels the voltage drop in that winding. This cancellation or compensation takes place in a uniform fashion so that the whole winding remains essentially at ground potential. This is achieved at the expense of an increased voltage drop in the uncompensated ratio winding but this can be tolerated since generally it has a lower number of turns. The potential difference between the two ratio windings is therefore less affected by compensation. This is one of the reasons why compensation of a comparator without electrostatic shield is less effective in reducing the capacitive error for large ratios.

The use of a compensation winding offers another advantage mainly concerned with the use of the device as a current-ratio standard for current-transformer calibrations. This will be discussed later. For the moment it suffices to point out that additional impedances can be connected in series with the outside winding of the compensated ratio winding without significantly affecting the current ratio of the comparator. Such an impedance is normally resistive and is used for producing small known deviations of the current ratio from its nominal value by a current-injection technique. Such deviations can be made direct reading.

CALIBRATION OF THE COMPENSATED CURRENT COMPARATOR AS A CURRENT RATIO STANDARD

Current comparators can best be calibrated using other current comparators in a build-up process. In general, three multi-ratio current comparators are required for this purpose. The main steps of such a process, using compensated current comparators, will be illustrated. The measurements involved are differential in nature so that the accuracy of the measuring circuit need not be very high (typically 1 per cent).

The error of a current comparator is defined by the following equation:

$$I_s = \frac{I_p}{n}(1 + \epsilon)$$

where

n = the turns ratio of the ratio windings
I_s and I_p = the currents in the ratio windings measured at their respective marked terminals when these terminals are at ground potential.

In the compensated current comparator the error ϵ can be considered the sum of two components: the zero burden error ϵ' and the error increase ϵ'' due to the burden Z. Burden is defined as the impedance obtained by dividing the voltage between the terminals of the secondary winding by the secondary current.

The error increase ϵ'' has been found to be proportional to burden, hence

$$\begin{aligned}\epsilon &= \epsilon' + \epsilon'' \\ &= \epsilon' + YZ\end{aligned}$$

where both ϵ' and Y are constants.

In the following calibration circuits the burden, if not zero, will be resistive (r) and equal to 0.1 Ω.

The calibration procedure starts at the 1/1 ratio. This ratio can be calibrated by the self-calibration circuit (Fig. 5). Note that the detection winding and associated tuned null detector have been omitted in order to simplify the diagram. The primary and secondary currents are compared directly and their difference is measured. Since the voltage drop across the compensation winding is small the desired potential conditions are very nearly realized. To bring the measuring terminals to exact ground potentials an auxiliary balance is required (detector D). This balance is most conveniently realized using a single turn through a magnetic core (K) carrying a control winding. (This auxiliary balance is common to all calibration circuits and will be omitted from the succeeding circuit diagrams.) The main balance consists in adjusting the conductance G and the capacitance C to bring the detection-winding voltage to zero. This leads to the reading δ of the measuring circuit, δ being defined as

$$\delta = (G + j\omega C)r$$

It is easily seen that for this circuit $\epsilon = \delta = (G + j\omega C)r$. It should be noted that this circuit also permits the calibration to be carried out with and without burden r. Hence ϵ' and ϵ'' can both be measured.

Once the 1/1 ratio error has been established the 2/1 ratio can be calibrated using either the $(N+1)$ or $(M+N)$ calibration circuit. In the $(N+1)$ circuit (Fig. 6) two current comparators are used with ratios of $n/1$ and $(n+1)/1$. Two current sources S_1 and S_2 are adjusted relative to each other so as to balance one of the current comparators. The other one is balanced by adjusting the measuring circuit δ which is identical to the one used in the self-calibration circuit. At balance

$$\epsilon_C = \delta + \frac{n\epsilon_B' - \beta_B}{(n + 1)}$$

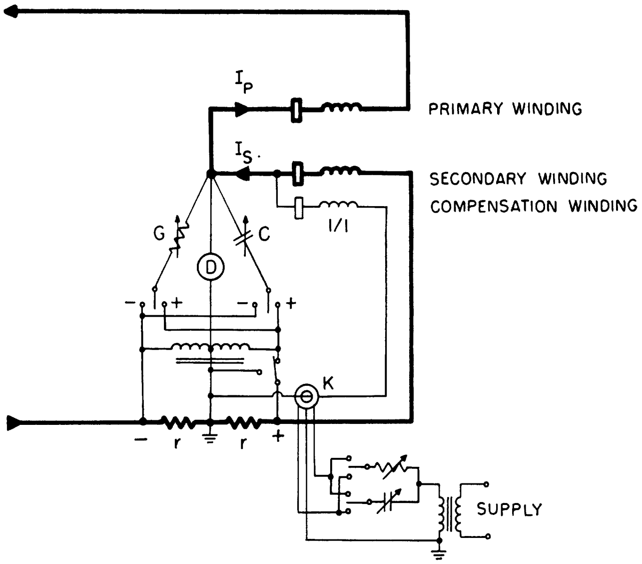
where β is defined as the proportional difference between the current at the unmarked terminal and the current at the marked terminal of the secondary winding at zero burden. This circuit makes possible the calibration of an $(n+1)/1$ ratio comparator in terms of an $n/1$ ratio comparator provided the factor β can be measured. β is measured using the circuit of Fig. 7. This is essentially the self-calibration circuit of comparator C with the secondary circuit of the comparator whose β coefficient is to be measured inserted in series. At balance

$$\beta_B = \delta - \epsilon_C$$

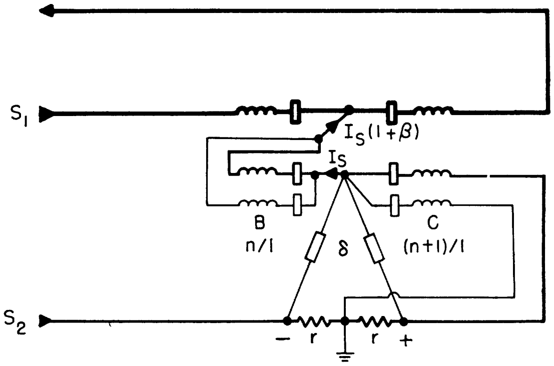
Using these two calibration circuits it is possible to calibrate a 2/1 ratio once the 1/1 ratio has been established by self-calibration. Once the 2/1 ratio is known, the 3/1 ratio can be established. By buildup in this fashion any ratio can be calibrated. This method has the advantage that only two multi-ratio comparators are required and only a double balance has to be achieved. The buildup process is however very slow and the multi-ratio comparators must be capable of being connected for all in-between ratios.

A faster buildup procedure is provided by the $(M+N)$ calibration circuit (Fig. 8). This circuit requires three comparators and a triple balance for each step. The primary currents of comparators A and B are added to form the primary current of comparator C. The secondary windings of all three comparators are connected in series as shown. The current sources S_1 and S_2 are adjusted relative to the common secondary current to null comparators A and B. Comparator C is then balanced by adjusting the measuring circuit δ.

Using this circuit a 2/1 ratio can be calibrated from two 1/1 ratios. Once a 2/1 ratio is established another 2/1 comparator can be calibrated either by rotation of the comparators in the $(M+N)$ circuit or by the comparison circuit of Fig. 9. Once two 2/1 ratio comparators are calibrated the errors of a 4/1 can be measured. The

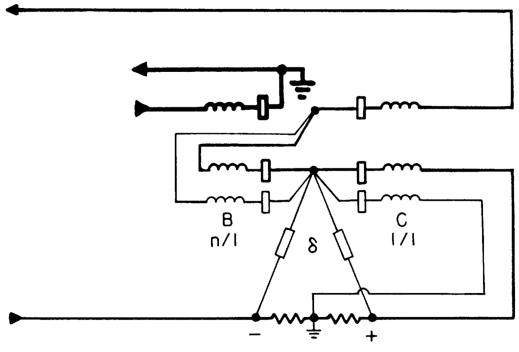


$\epsilon = \delta = r(G + j\omega C)$.
Fig. 5. Self-calibration circuit.



$$\epsilon_C = \delta + \frac{n\epsilon_B' - \beta_B}{(n+1)}$$

Fig. 6. $(N+1)$ circuit.



$\beta_b = \delta - \epsilon_C$.
Fig. 7. β circuit.

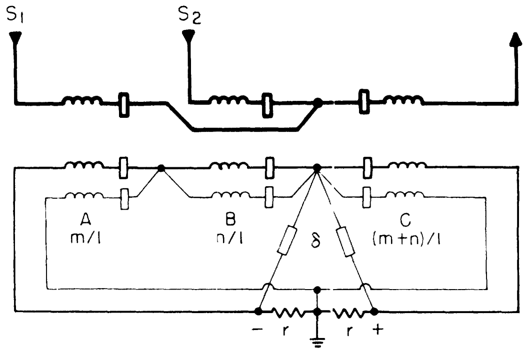
buildup sequence can be as follows:

1/1 1/1 2/1 2/1 2/1 4/1 4/1 4/1 8/1 8/1 8/1 16/1

As mentioned before the buildup procedure for large ratios is much quicker and not all in-between ratios are required.

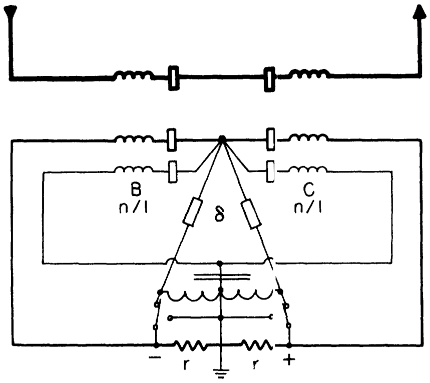
An even faster buildup to large ratios can be realized using the cascade-calibration circuit (Fig. 10). In this circuit two comparators of ratios $m/1$ and $n/1$ are connected in cascade to provide the reference for a $mn/1$ ratio. An additional advantage of this circuit is the absence of the β term in the balance equation. For comparators without an electrostatic shield between the ratio windings this β term can be quite large at the higher-audio frequencies and thus become the factor limiting the accuracy of any calibration involving this term. The disadvantage of the cascade circuit is that not all current comparators are operated at their rated current so that only a lower sensitivity is available.

A feature common to all calibration circuits is that, in addition to the potential balances which are auxiliary in nature, there are two or three main detection winding balances to be achieved. Each of these main balances are affected not only by the adjustment of the measuring circuit but also by the adjustment of the current supplies S_1 and S_2. The final setting of the measuring circuit depends only on the error characteristics of the comparators which are very stable. This stability is however obscured by any instability in the current supplies S_1 and S_2. This effect can be eliminated by adding in a suitable fashion the detection winding voltages to produce a final balance indication which is not affected by drifts in the current supplies and depends only on the measuring circuit adjustment [4]. This result can be achieved readily at any one frequency and contributes markedly to the precision of the measurement. Unfortunately, the procedure is frequency sensitive and this limits its value when covering a large frequency range.



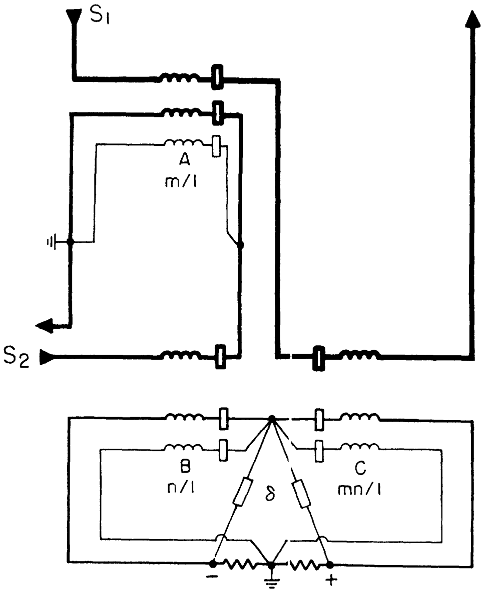
$$\epsilon = \delta + \frac{m}{m+n}(\epsilon_A - \beta_B) + \frac{m}{m+n}\epsilon_B'.$$

Fig. 8. $(M+N)$ circuit.



$$\epsilon_C = \delta + \epsilon_B.$$

Fig. 9. Comparisor circuit.



$$\epsilon_C = \delta + \epsilon_A' + \epsilon_B.$$

Fig. 10. Cascade circuit.

The buildup calibration which has been described can be called "absolute" since no assumptions are involved Each current comparator is treated as a "black box" whose characteristics are measured at its terminals. Nothing needs to be known about its internal construction. If, however, the internal construction is known an alternate calibration procedure is available. This procedure consists in computing the errors. First the magnetic error is stablished. In a well-designed current comparator it will be negligibly small. To determine this experimentally it is sufficient to establish a balance in a comparison circuit and then purposely magnetize the core with a dc current. The shift in balance will be an indication of the magnetic error. If it is indeed negligibly small the balance should not shift. The only observable effect will be a decrease in sensitivity. The only remaining error then is the capacitive error and this error can be calculated. The accuracy of such a calculation depends on the starting point and the validity of the assumptions made. For comparators designed to achieve uniform capacitance distribution it has been found that such calculations, based on measured values of voltage drops and capacitances, and on the assumption of perfect uniformity of capacitance distribution, agree with the results of the absolute calibration to about 1 ppm at 16 kc. Since this method is independent of ratio, it is very powerful for very large ratios where the absolute calibration is very long and tedious and also less accurate because of error accumulation in the buildup process.

Calibration Results

It is not within the scope of this paper to give detailed calibration results and corresponding constructional information about the different current comparators which have been built. Some limited information will however be useful to illustrate the order of magnitude of the errors which can be realized in the audio-frequency range.

It was decided from the outset that several compensated-current comparators of different design would be constructed in order to compare their performance. All of these current comparators have identical magnetic cores, detection windings and magnetic shields and essentially the same number of turns in their ratio windings. They differ, however, in the geometry of the ratio windings. In Type I the ratio windings are in a single layer and are interleaved to minimize turn to turn capacitance. In Type II the ratio windings are in two layers and are not interleaved. Each ratio winding occupies one layer and the two layers are spaced to reduce the winding-to-winding capacitance. In Type III the winding-to-winding capacitance is reduced to zero by the use of a copper shield between the ratio windings. Ratio variation in each type is obtained by series-parallel connection of the primary windings.

Typical error characteristics for zero burden are illustrated in Fig. 11. Of particular interest is the magnitude of the error, and the error variation with ratio. The following conclusions can be drawn:

1) The 1/1 ratio errors are about the same for all three types.
2) Only Type III is characterized by a reduction of the error as the ratio is increased.

Type III appears best suited for large ratios. A 100/1 ratio comparator of this design has been constructed and calibrated. Its errors are less than 1 ppm at all frequencies up to 16 kc/s.

APPLICATIONS OF THE CURRENT COMPARATOR

Calibration of Current Transformers

The ratio of a current transformer must be clearly defined before it can be measured with precision. It is convenient to define this as the complex ratio of the currents entering and leaving the windings at two terminals of the same polarity when both these terminals are at ground potential. Calibration consists in the measurement of the deviation of this ratio from its nominal value n. This deviation is the error ϵ_T of the current transformer and is defined by the following equation:

$$I_s = \frac{I_p}{n}(1 + \epsilon_T)$$

In general ϵ_T depends on burden and current.

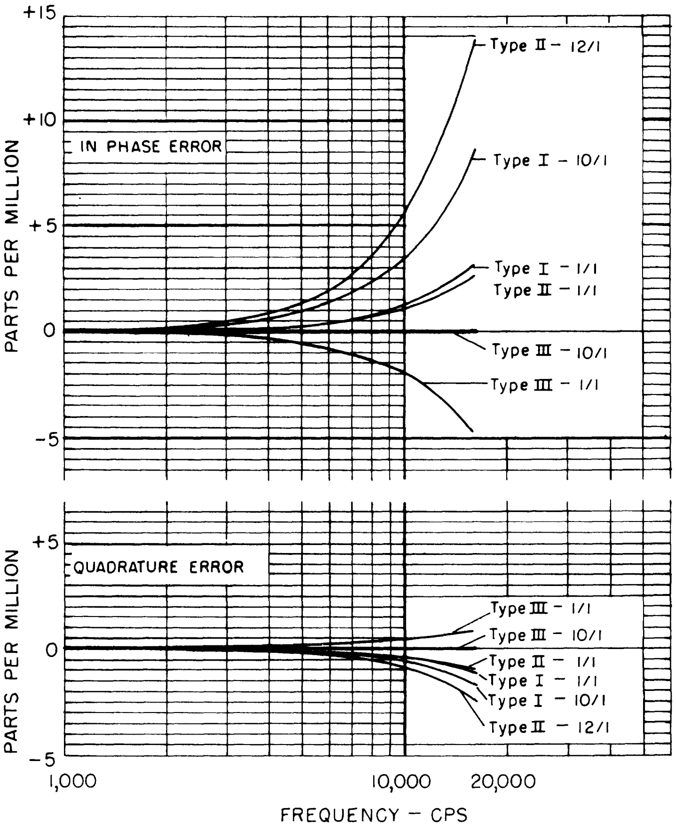


Fig. 11. Typical zero-burden error characteristics of audio-frequency compensated-current comparators.

The calibration of a current transformer can best be carried out by comparing it to a current comparator whose turns ratio is equal to the nominal ratio of the current transformer. If the current comparator is uncompensated a deviation winding is required on the comparator (Fig. 12). Two auxiliary balances are necessary to bring the measuring terminals to ground potential (using detector D_1 and D_2). The main balance consists in adjusting the conductance G and the capacitance C to bring the detection winding voltage to zero. The balance equation is as follows:

$$\epsilon_T = (G + j\omega C) r \frac{N_d}{N_s} + \epsilon$$

The error ϵ of the comparator is usually negligible.

This circuit has the disadvantage that the burden on the current transformer cannot be reduced to zero. The secondary of the current comparator and the measuring resistors r constitute a minimum burden on the current transformer. This minimum burden may be excessive, particularly at power frequency operation where the resistors r have to be relatively high in order to obtain a sufficiently large range of the measuring circuit with reasonably sized capacitor C components.

These difficulties can be overcome when a compensated-current comparator is used as the standard (Fig. 13). Here the magnetic shield of the comparator together with the outside ratio windings act as a current

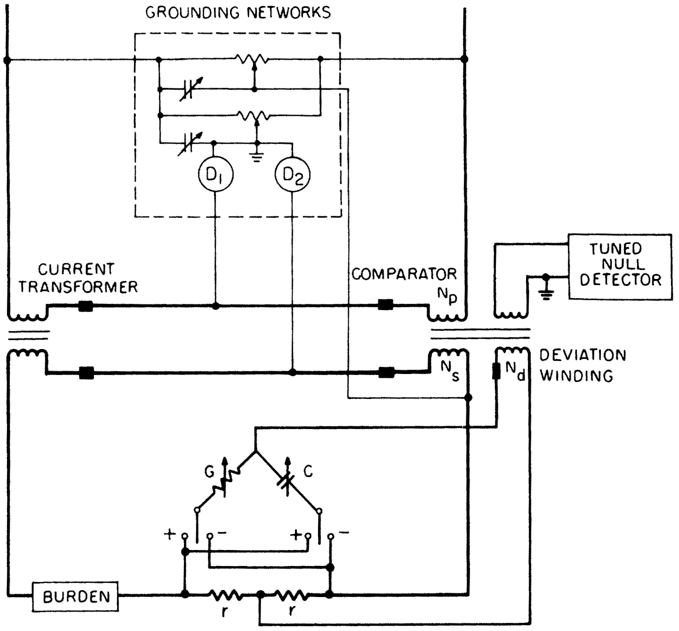


Fig. 12. Current-transformer calibration circuit using uncompensated current comparator.

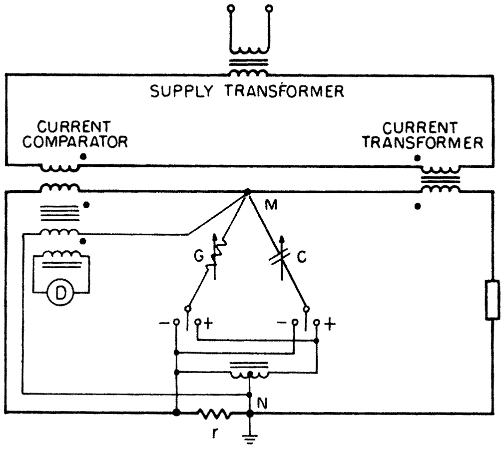


Fig. 13. Current-transformer calibration circuit using the compensated-current comparator. (Primary feed.)

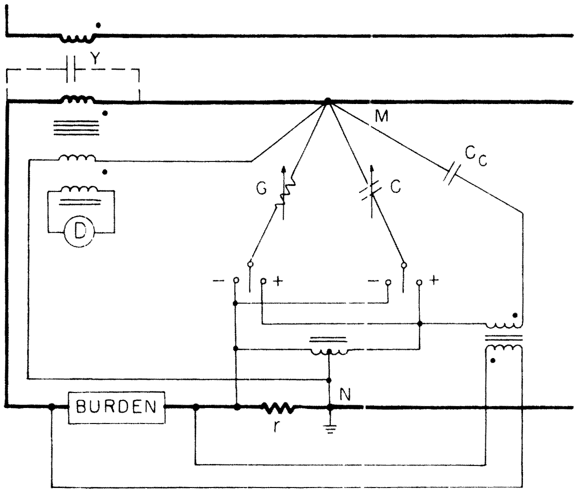


Fig. 14. Current comparator with correction for error variation due to burden.

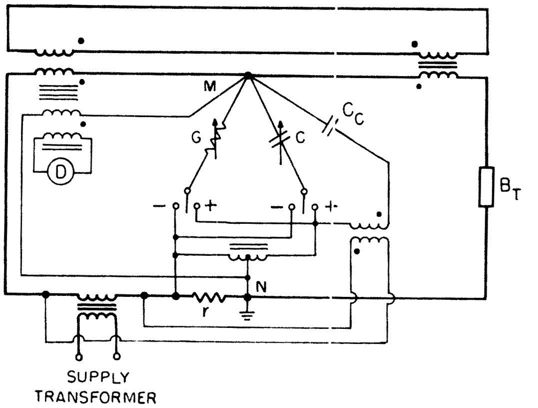


Fig. 15. Current-transformer calibration circuit using the compensated-current comparator. (Secondary feed.)

transformer which carries the minimum burden previously considered. The residual burden imposed by the comparator on the current transformer is very low (typically less than 1 mΩ) and can usually be neglected. If required it can be eliminated by an auixiliary balance using a single-turn control transformer as in the self-calibration circuit (Fig. 5).

It should be noted that the above advantages can be realized only if the secondary ratio winding of the comparator is compensated. The reduction of the capacitive error by compensation requires the winding with the largest number of turns to be compensated. It is a fortunate coincidence that most current transformers have a nominal ratio larger than one, where both requirements are identical. This is not so with stepup current transformers.

The calibration of a curent transformer using a compensated-current comparator as a standard is a simple single-balance operation. If the error of the standard can be made negligibly small, the circuit can be made direct reading. This condition can be realized quite readily at the power frequencies since the capacitance error is small and the magnetic error can be practically eliminated by magnetic shielding. In actual practice the small capacitive error, which is linear and depends mainly on the burden can be overcome by a correction network illustrated in Fig. 14. When corrected in this fashion the error of the current comparator is independent of burden and the burden can be increased to the point where saturation starts to occur in the magnetic shield. If the magnetic shield is made sufficiently large in cross section to carry the volt-amperes required in the large-current supply-transformer, the latter can be dispensed with, using the calibration circuit of Fig. 15. In this circuit the secondaries are series connected with the power supply transformer and the only current supply required to carry out the calibration is a 5-A rated adjustable autotransformer. The shield fulfills the double function of a magnetic shield on the one hand and the core of the supply transformer on the other.

Impedance Comparison

Fundamentally, there are only two methods of comparing two impedances or admittances: the same voltage is applied to both and the current ratio is measured, or the same current is made to flow in both and the voltage ratio is measured. A combination of both methods is also possible. The current comparator is ideally suited to make impedance comparisons based on the first method.

Any impedance comparison based on the current comparator is a double balance operation: a voltage-equality balance and a current-comparison balance. For very large impedances however the voltage equality balance can be made self-generating [Fig. 16(a)]. This prevails as long as the effective winding impedances can be neglected with respect to the impedances to be compared. A typical application is the high-voltage capacitance bridge [8] [Fig. 16(b)]. This bridge will be recognized as the conjugate of the three-winding voltage-ratio-transformer bridges used by Thompson [6] and McGregor, et al [9]. In comparison, the current-comparator bridge is less sensitive because of the difficult match which has to be realized between the current comparator and the capacitances to be measured. The bridge finds its proper application field in high-voltage measurements where sensitivity requirements are not too restrictive.

When the effective winding impedance can no longer be neglected a second balance is required to assure equality of the voltage drops across the two impedances [Fig. 17(a)]. Both balances are of equal importances which is an undesirable situation. When magnetic shielding is used in the comparator a compensation winding can be used to make the voltage balance nearly automatic [Fig. 17(b)]. This balance then becomes a secondary balance and the operation of the bridge is greatly facilitated. For this purpose the number of turns of the compensation winding should be equal to the sum of the number of turns of the two ratio windings. The fact that the current terminals of the comparator are independent of the voltage-balance terminals of the compensation winding makes this bridge suitable for the comparison of three-terminal impedances. When four-terminal impedances are to be compared an additional voltage balance at the other voltage terminals of the impedances is essential (Fig. 18). This balance is only a lead impedance balance and consequently of secondary importance.

This bridge will be recognized as the conjugate of the voltage-ratio-transformer bridge described by Gibbings [10]. It is to be noted however that the conditions, as far as the resistors are concerned, are quite different in these two bridges. In Gibbings' bridge the resistors carry the same current, consequently when resistors of unequal value are compared the largest power dissipation occurs in the highest value resistor. In the current comparator bridge, the same voltage is applied to the resistors so that the highest power dissipation will occur in the lowest value resistor. When the effect of power dissipation on the value of the resistors cannot be neglected, the two bridges will give different answers. The current comparator bridge is suited for the calibration of low value shunts at high currents. In this bridge the shunts can be operated at full current without surpassing the limits of power dissipation in the higher value reference resistor. The fact that large ratios can be realized in current comparators with high precision is a very important factor in assuring that the power dissipation in the reference resistor is kept below the tolerable level.

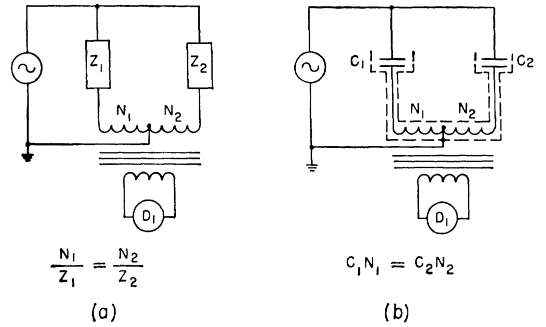


Fig. 16. Bridges for the comparison of high impedances.

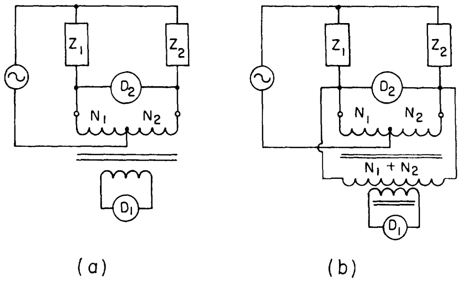


Fig. 17. Bridges for the comparison of low-value three-terminal impedances.

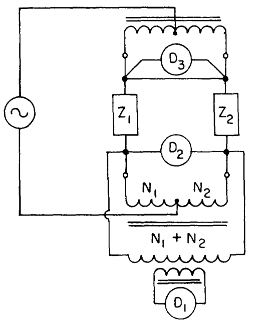


Fig. 18. Bridge for the comparison of low-value four-terminal impedances.

The DC Current Comparator

When the operating frequency of the current comparator is decreased, the sensitivity decreases, becoming zero at zero frequency. It shares this characteristic with its conjugate—the three-winding voltage-ratio transformer. This limitation cannot be overcome in the voltage-ratio transformer since the core is used to transfer power. In the current comparator however no energy transfer takes place and the magnetic balance condition in the core can be detected [11]. This is best accomplished by modulating the permeability of the sensing core. For this purpose the sensing core is made up of two separate cores (Fig. 19) which can be magnetized to saturation in opposite directions by an ac modulation current. Twice every cycle, when saturation occurs, the permeability of the composite core drops to a low value. When unsaturated, the permeability is high. If this variation of permeability could be made sinusoidal, a dc ampere-turn unbalance would induce in the detection winding a sinusoidal voltage of second-harmonic frequency; hence the name: second harmonic modulator. The actual variation in permeability produced will not be sinusoidal. With high permeability core material the transition from the unsaturated to the saturated region will be fairly sharp and the corresponding induced voltage will be in the form of sharp spikes occurring every half cycle. The main component of this induced voltage will still be of second harmonic frequency but additional even frequency harmonics will also be present. With the aid of a tuned filter, the second harmonic induced voltage can be separated from the other components and used as a dc ampere-turn balance indicator.

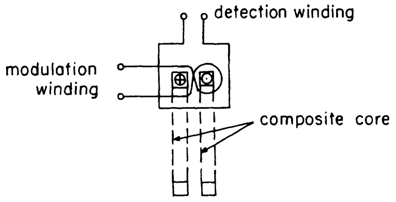


Fig. 19. The second-harmonic modulator.

There are however limitations to this technique. It has been found that considerable noise is introduced into the detection winding by the modulation circuit. This noise has two components, 1) one in quadrature with the signal produced by an ampere-turn unbalance and 2) one in phase. The first prevents the achievement of a perfect balance and reduces the sensitivity to zero near balance. The second represents an error in the balance. The effect of the first can be eliminated by the use of a phase-sensitive second-harmonic detector. When this is done the detector output becomes a dc signal whose polarity reverses with the polarity of the ampere-turn unbalance.

Another type detector producing the same type of output is the peak detector. Such a detector compares the positive and negative peaks of the unfiltered detection-winding voltage and provides an output proportional to their difference. The output of such a detector will be zero for a sinusoidal signal of any frequency. When more than one frequency is present however, such as a residual fundamental-frequency voltage due to core unbalanced and a second-harmonic frequency voltage due to an ampere-turn unbalance, the detector will indicate the magnitude and polarity of this ampere-turn unbalance. This detector is easy to construct and has been found very satisfactory.

The sensitivity of these devices can be made exceedingly high. The highest usable sensitivity however is limited by noise as reflected in their zero stability. This stability displays two main characteristics:

1) Memory effect. The zero is different after a heavy overload. This characteristic is important when such heavy overloads cannot be avoided as may be the case in industrial applications. For laboratory operation this effect is of minor importance in much the same way as a zero shift produced by a heavy overload on a galvanometer.
2) Spontaneous zero drift. This is the important one, as far as laboratory operation is concerned, and limits the sensitivity that can be usefully employed in any experiment.

Both these characteristics are very dependent on the intensity of the modulation applied to the cores. The zero stability is best when the cores are driven deep into saturation. When this is done the spontaneous zero drift can be reduced to less than 10-μA turns for core diameters as large as seven inches.

The ratio accuracy of the dc current comparator is free from capacitive error. The corresponding error due to insulation leakage resistance presents no problem with modern insulating materials. The only remaining error is the magnetic error. The use of magnetic shielding has been found just as effective in reducing the magnetic error at zero frequency as at the higher audio frequencies. The adequacy of a particular shield can be assessed by the same methods. Experience at NRC has indicated that ratio accuracies of 1 ppm are readily obtainable for any ratio between 1/1 and 1000/1.

THE APPLICATIONS OF THE DC CURRENT COMPARATOR

The Calibration of Transductors or DC Current Transformers

Experience at NRC with transductors or dc current transformers is limited. Their ratio depends on current and burden as in ac current transformers but in addition it depends on the amplitude and possibly the waveshape of the excitation voltage and the position of the primary conductors. It appears however that repeatability to well within one part in ten thousand of rated current can be realized in practice. The calibration of such devices to an accuracy beyond their repeatability is well within the capability of the dc current comparator.

The calibration of a transductor is similar to the calibration of an ac current transformer and similar circuits can be used. The use of the magnetic shield for energy transfer to supply the losses in the secondary winding of the comparator is no longer possible however and these losses have to be supplied by other means.

The circuit of Fig. 20 has been found to be very convenient for this purpose. The current comparator has one primary-ratio winding and a composite secondary-ratio winding consisting of three windings having the same number of turns: the bias winding, the feedback winding and a ripple-suppression winding. A double balance has to be achieved: a voltage or burden balance as indicated by the galvanometer g and an ampere-turn balance on the comparator.

In principle only one secondary-ratio winding, the bias winding, is required. The power supply S is adjusted to null the galvanometer g, the conductance G is varied to null the comparator detector. At balance

$$\epsilon_T = rG$$

To be able to achieve this double balance, exceedingly stable power sources are required. This is nearly impossible to achieve, even in the laboratory. This difficulty can be overcome by making one of the balances automatic by the use of a feedback amplifier and the feedback winding. If the current-comparator balance is made automatic the comparator becomes self-balancing. This is the solution chosen for some of the NRC designs. It has the advantage of preventing magnetic memory errors and greatly increases the speed of response of the comparator detector. It is however also possible to make the voltage balance automatic and this approach presents advantages in some applications.

In practice the primary current will not be free of ripple. This ripple has to be reproduced in the secondary ratio winding. The fidelity of this reproduction is degraded however by the internal impedance of the power supply S and the output impedance of the feedback amplifier. For this reason either or both these internal impedances have to be very low. To eliminate this requirement it is very often convenient to use a ripple suppression winding. This winding acts in conjunction with the magnetic shield to form an ac current transformer. The function of capacitor C is to eliminate any ac coupling between the comparator and the transductor.

Most devices used for the measurement of large dc currents are affected by ambient conditions and are best calibrated *in situ*. Figure 21 illustrates a 20 000 ampere comparator designed for this purpose. It is self-balancing up to ±2000 ampere turns. The feedback loop gain is larger than 10 000 so that at 20 000 A its error is less than one part in 10^5. Its resolution is better than 1 ppm at full current. Its accuracy at rated output is better than one part in 10^5 for any configuration of the primary conductors. The largest ratio available is 2000/1 but larger ratios, up to 20 000/1, can be obtained by cascading it with a second self-balancing comparator of 10/1 ratio. The accuracy performance is not degraded by the use of the second comparator provided the second comparator is not located too close to the strong magnetic field of the primary circuit.

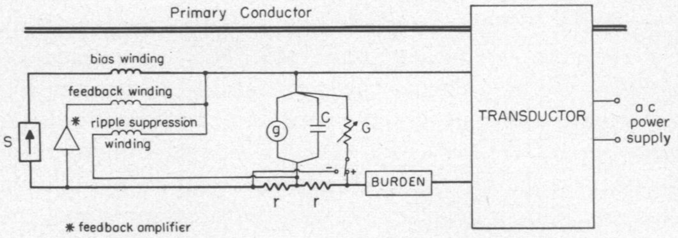


Fig. 20. Transductor calibration circuit.

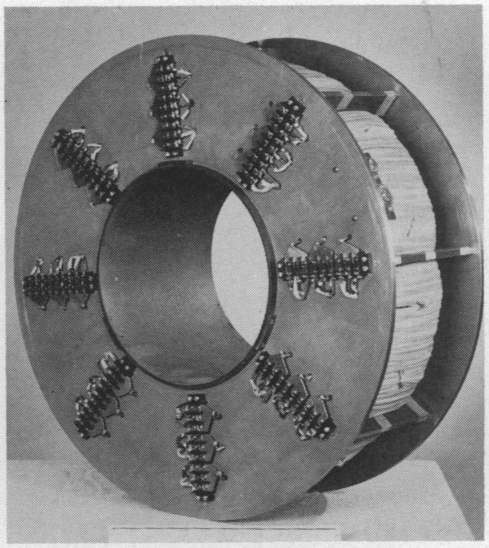


Fig. 21. 20 000-A dc current comparator.

The Calibration of DC Shunts at Full-Rated Current

The use of shunts for the measurement of large dc currents has always looked attractive because of its simplicity and the high resolution that can be obtained readily with the aid of a potentiometer. It is often overlooked however that accuracy and resolution are not one and the same thing. A well-designed shunt can give good repeatability. The accuracy of the current measurement, however, is limited by its calibration.

It is common practice to calibrate shunts at a low current and to determine their temperature dependence. It is left up to the user of the shunt to measure the operating temperature and thus determine its operating resistance. This would be quite acceptable if the shunt temperature were uniform and thus easily measurable. In practice this condition is not always present. The only temperature that can be made uniform and easily measurable is the temperature of its environment such as an oil bath. If the circulation in the oil bath is sufficient the temperature gradient within it can be kept below a tolerable limit. The actual temperature of the resistance alloy however will not necessarily be the same as the oil bath. For this reason it is better to calibrate shunts at full current.

Shunts are calibrated using conventional bridges by comparing their voltage drops to the voltage drop across a standard carrying the same current. The resistance value of the standard must be established by calibration in terms of the standard ohm which cannot carry the large currents involved. It can however, stand the same voltage drop. The current camparator can establish the current ratio corresponding to voltage drop equality and so measure the resistance ratio at full current.

Not much work has been done in this field. Some results can be given however. These concern the comparison of a 1-ohm resistor and a 1/100-ohm resistor, both of standard quality. Both were submersed in the same oil bath. The oil bath was water cooled, with a heater and thermostat to maintain the temperature of the oil constant. The agitation was such that no temperature rise could be measured in the thermometer pockets of the resistors.

Figure 22 illustrates results obtained. The resistance ratio varied as much as 50 ppm. This was due mainly of the heating of the 0.01-ohm resistor. The ratio at low currents corresponded within 2 ppm to the ratio established by conventional bridge methods.

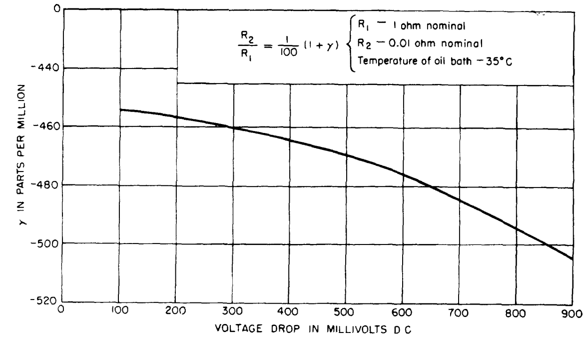


Fig. 22. Resistance-ratio variation with current.

CONCLUSION

The current comparator has been found to be a very accurate, stable and linear current ratio standard. By careful design of the ratio windings it can be constructed to have these characteristics without the use of a magnetic shield provided magnetization of the core is avoided. Its full potentialities however become apparent only when it is equipped with a magnetic shield. When this is done, its ratio becomes practically independent of the magnetic condition of the core and careful design of the ratio windings becomes less important, particularly at the lower audio frequencies. The deviation of its ratio from the turns ratio is then mainly of capacitive origin and even this source of error can be reduced considerably by the magnetic shield through the use of a compensation winding.

For ac operation the use of the current comparator is limited only by the noise in the detector and a significant improvement in sensitivity can be achieved with large core cross sections of high initial permeability material. For dc operation the main source of noise is of magnetic origin [12] and little improvement in usable sensitivity can be expected from large-core cross sections.

The use of the current comparator for the calibration of other ratio devices, such as current transformers, is also greatly facilitated by the magnetic shield and a compensation winding. It would appear that similar techniques for the calibration of voltage-ratio transformers are worth investigating.

ACKNOWLEDGMENT

The work reported in this paper is not the work of one man but the coordinated effort of a team. It has been a privilege to be part of that team. The cooperation of W. J. M. Moore, O. Petersons, M. P. MacMartin and P. Miljanic is hereby gratefully acknowledged.

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