

A Direct-Current-Comparator Ratio Bridge for Four-Terminal Resistance Measurements

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Abstract—A ratio bridge has been built which compares four-terminal resistances by measuring the current ratio corresponding to voltage drop equality. A current ratio, corresponding to turns ratio, is automatically maintained by a self-balancing direct current comparator and is adjustable in part-per-million steps. The two current sources are isolated so that, at balance, there is no current in the potential circuit. The bridge can be direct reading in either resistance or conductance ratio.

The bridge makes use of the ability of a self-balancing direct current comparator to measure and compare accurately the ratio of two isolated direct currents. A detailed description of the bridge is presented with an analysis of its accuracy limitations. In its optimum operating range accuracies of better than one part per million can be achieved.

The bridge is particularly suited to the scaling of resistance standards from one hundred ohms to a fraction of a milliohm. It can be used at any ratio up to one thousand to one. It permits the calibration of low value shunts at full operating current with an accuracy limited only by their noise level and stability.

INTRODUCTION

IN THE LAST decade, considerable progress has been made in alternating current impedance measurements through the use of ratio transformers [1]–[8]. The excellent ratio stability of these devices, together with their high magnetizing and low leakage impedances, has led to new methods for comparison and scaling of impedance and admittance standards. No similar progress has been achieved, however, in the measurement of direct current resistors, which form the basis of our system of units. The bridge described in this paper is an attempt to introduce the advantages of ratio transformers to direct current resistance measurements.

A four-terminal resistance is defined as the ratio of the voltage between its potential terminals to the current passing through its current terminals when no current passes through its potential terminals. Four-terminal resistances can be compared by measuring either the voltage ratio corresponding to current equality or the current ratio corresponding to voltage equality.

In the potentiometric method the two resistors to be compared are connected in series to ensure current equality and the corresponding voltage drops are each measured separately by a potentiometer [Fig. 1(a)]. The main advantage of this method is that the measurement is completely free from any effect of potential lead

resistances. Its main disadvantage is that it requires a current ratio stability between the resistor circuit and the potentiometer circuit which is at least as good as the resolution required in the measurement. When this resolution approaches one part per million (ppm) the corresponding current ratio stability is often difficult if not impossible to achieve since it is affected by both battery drift and lead resistance variations. The lead resistances concerned are those in the current circuit where they may constitute an appreciable portion of the total circuit resistance.

A significant improvement in the potentiometric method was introduced by T. M. Dauphinee with his "Isolating potential comparator," [9] [Fig. 1(b)]. This device generates the difference between the potential drops in the two resistances without making a direct differential connection. The potentiometer measures only this difference and the current ratio stability required has thus been reduced considerably. This advantage is most pronounced for one to one comparisons. It almost disappears for scaling of resistance standards.

In the Kelvin Double Bridge method [Fig. 1(c)] the same current is again made to flow in the two resistors by connecting them in series, but the voltage ratio is determined directly by resistive voltage dividers connected to the potential terminals. This connection cannot be made, however, without a current drain from the

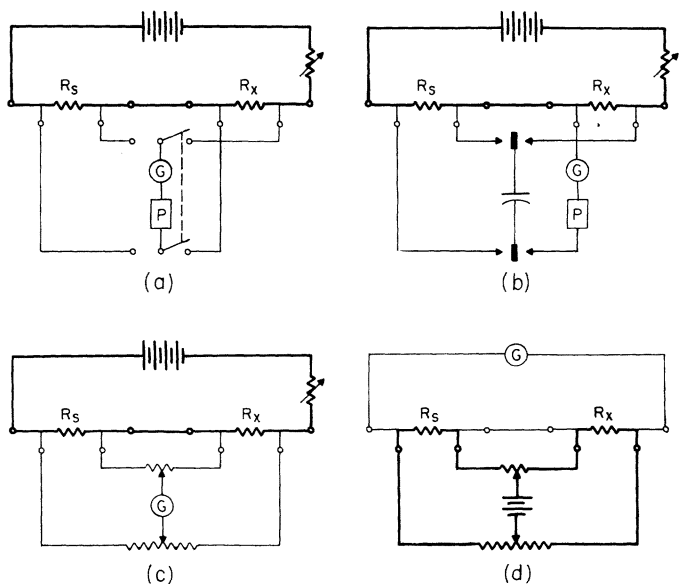


Fig. 1. Conventional measurement methods. (a) Potentiometric method. (b) Isolating potential comparator method. (c) Kelvin Double Bridge. (d) Conjugate of Kelvin Double Bridge.

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potential terminals and additional auxiliary balances are required to eliminate the effect of potential lead resistances. The adverse effect of battery drift is completely eliminated, but lead resistance variations still affect the result. The lead resistances concerned are in the potential circuit, however, where the total resistance level can be made sufficiently high to reduce the effect to a tolerable level.

It is not obvious why measurements made on the Kelvin Double Bridge correspond to the actual resistance ratios since the potential terminals are not completely current free as required by definition. This point can be clarified by considering the conjugate bridge [Fig. 1(d)], obtained by interchanging the battery and the galvanometer. In this configuration it is evident that one pair of terminals is actually current free. Since the reciprocity principle is applicable to the bridge and to the two resistors to be compared, it follows that the bridge does indeed measure the true resistance ratio.

In the conjugate of the Kelvin Double Bridge, currents which are adjustable in ratio are made to flow in the two resistors to establish voltage equality. At balance, no current flows in the galvanometer potential leads but the condition that no current flows in the other potential leads has to be established by an auxiliary balance. This auxiliary balance could be eliminated if the two currents were derived from isolated batteries instead of from the same battery, and a device were available which could measure the ratio between the currents in the two isolated circuits. The current comparator is such a device [10], and its application to the comparison of four-terminal resistances is described in this paper.

THE CURRENT COMPARATOR

The current comparator is a current ratio indicator based on the detection of a zero flux condition in a magnetic core. For alternating current operation, the detector consists of a single magnetic core with a uniformly distributed detection winding; for direct current operation, a double core magnetic modulator is used [11], [12].

The main components of a direct current comparator are

- 1) a double toroidal core magnetic modulator,
- 2) a magnetic shield which completely surrounds the modulator cores, and
- 3) two ratio windings which link both the modulator and the magnetic shield and which carry the currents to be compared.

When the modulator output is zero the current ratio is equal to the turns ratio to a high degree of accuracy.

The purpose of the magnetic shield is two-fold. It increases the accuracy of the device by making it insensitive to ambient magnetic fields, either internally or externally generated. It also reduces the coupling between the ratio windings and the modulator. The sensitivity

of the modulator can thus be made nearly independent of the impedance level in the ratio windings. A shield of high permeability magnetic material is very desirable.

When the number of turns of one of the ratio windings is made adjustable, the direct current comparator can be used to measure the current ratio in two isolated circuits. When the turns are adjusted to null the detector, the turns ratio corresponds to the current ratio. This ratio can readily be made direct reading.

THE DIRECT-CURRENT-COMPARATOR RATIO BRIDGE

The direct-current-comparator ratio bridge is shown schematically in Fig. 2. In its elementary form it consists of two isolated, manually adjustable, electronically regulated power supplies feeding current to the two resistors to be compared through the two ratio windings of the current comparator. Balance is achieved when both the galvanometer and the magnetic modulator output are at zero.

At balance

$$\frac{R_x}{R_s} = \frac{G_s}{G_x} = \frac{I_s}{I_x} = \frac{N_x}{N_s} .$$

If N_x is made adjustable the bridge is a direct reading resistance bridge

$$\left(R_x = N_x \frac{R_s}{N_s} \right) .$$

If N_s is made adjustable the bridge is a direct reading conductance bridge

$$\left(G_x = N_s \frac{G_s}{N_x} \right) .$$

Bridge balance is indicated by two simultaneous balances: a voltage balance and a current ratio balance. Two controls are provided to achieve this condition: turns ratio and relative power supply setting. To achieve this balance in the elementary bridge extremely high stability in the current ratio and resolution in the power supply settings are required. These conditions are nearly impossible to realize, but even if realizable the bridge

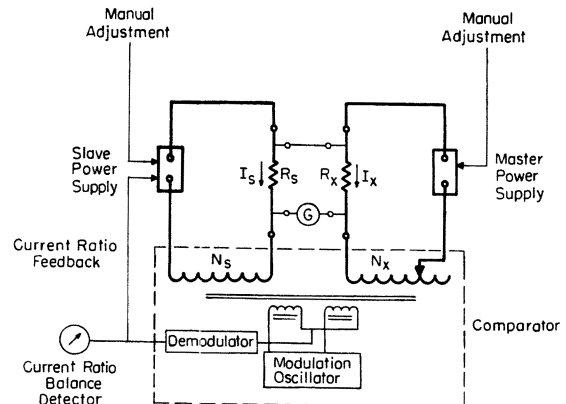


Fig. 2. Direct-current-comparator ratio bridge.

would be difficult to balance because of the interaction between the two controls. Reduction and possibly elimination of these limitations could be achieved by adding the two null detector outputs in appropriate proportions as indicated by Thompson [6]. This adding operation is, however, difficult to realize without destroying the isolation between the two current circuits. In addition, a second balance operation to be carried out consecutively with the main balance would be required. A better solution is to make one of the main balances semi-automatic by feedback. Either balance can be chosen for this purpose [12], but the current ratio balance, as indicated in Fig. 2, is the only one which does not destroy the isolation between the two current circuits. The current comparator thus becomes a self-balancing current comparator [13], which is a current ratio device having essentially the same operating characteristics as an alternating current transformer, but whose operating frequency range has been extended to include direct current. This technique introduces, however, a limitation in the number of turns N_s on the standard side which, for proper operation of the feedback loop, should be nearly constant and as high as possible.

DESIGN CONSIDERATIONS AND CONSTRUCTIONAL DETAILS

A. Ratio Windings

Experience has shown that, if a comparator is built without a magnetic shield, reasonable accuracy can be obtained only if the ratio windings are uniformly distributed and tightly coupled. The magnetic shield however eliminates these requirements almost completely. The ratio winding design can then be based on such considerations as direct reading features, ease of operation and ratio range.

For practical reasons, the largest number of turns in any one ratio winding is about a thousand. This provides a maximum resistance ratio of one thousand to one which is quite adequate. The corresponding per turn resolution of the bridge of one in a thousand is too low. To extend this to one in a million it is necessary to split one turn into a thousand parts. This can be achieved with satisfactory accuracy and stability by a resistive divider network.

The ratio windings are illustrated in Fig. 3. One ratio winding has a fixed number of turns (1000), the other has an adjustable number of turns (0–1000). Two resistive divider networks, marked ϵ_x and ϵ_s , are made available to provide fractions of a turn. One of these is always connected in series with the fixed, one-thousand-turn winding. The other can be switched to either ratio winding provided the operating current is within its current carrying capacity (one ampere).

The current capacity of the ratio windings is limited by the switches and is about one ampere. For operation at higher currents additional windings of 100, 10, and one turn with higher current capacities are provided as indicated in Fig. 3.

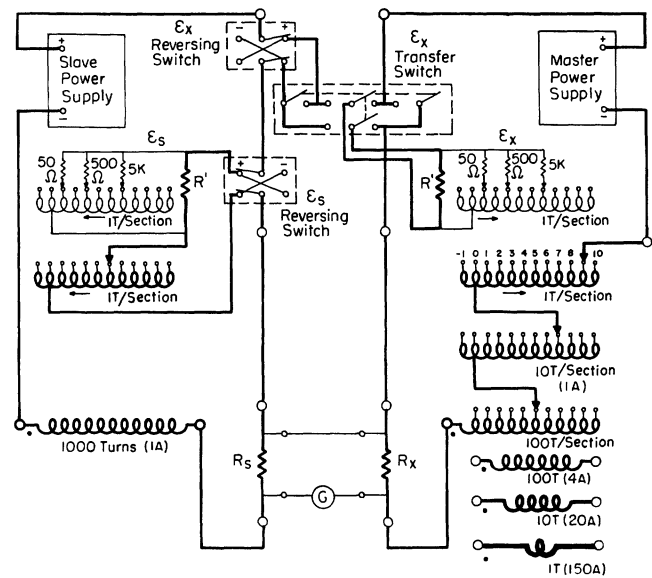


Fig. 3. Bridge ratio windings.

The bridge can be direct reading for either resistance or conductance. However, since the number of turns on the feedback side should remain high and nearly constant the range of the conductance bridge is limited.

For direct reading resistance measurements the fractional turns marked ϵ_x are connected in series with N_x , and N_s is made equal to 1000 (by setting ϵ_s to zero). The bridge is then a full range, direct reading ratio bridge as indicated by the balance equation:

$$R_x = N_x \frac{R_s}{1000}.$$

If ϵ_s is set equal to the deviation from nominal of the standard resistance the bridge then becomes direct reading in ohms. By definition

$$R_s = R_{sN}(1 + \alpha_s) \quad \text{and} \quad N_s = 1000(1 + \epsilon_s).$$

Setting $\epsilon_s = \alpha_s$, then

$$R_x = N_x \frac{R_s}{1000(1 + \epsilon_s)} = N_x \frac{R_{sN}(1 + \alpha_s)}{1000(1 + \epsilon_s)} = N_x \frac{R_{sN}}{1000}.$$

These two operating modes have ppm resolution only for one-to-one comparisons. When the unknown resistance is ten times smaller than the standard, the resolution of the bridge is reduced by a factor of ten and this becomes even coarser for larger resistance ratios.

To overcome this problem the fractional turns marked ϵ_x can be switched to the standard side. The nominal resistance ratio can then be set to three significant figures with the remaining N_x dials. The deviation from nominal of the standard (α_s) is set in the ϵ_s dials and the bridge is balanced by varying the ϵ_x dials. In this mode the bridge is then direct reading in deviation from nominal of the unknown. This may be shown as follows:

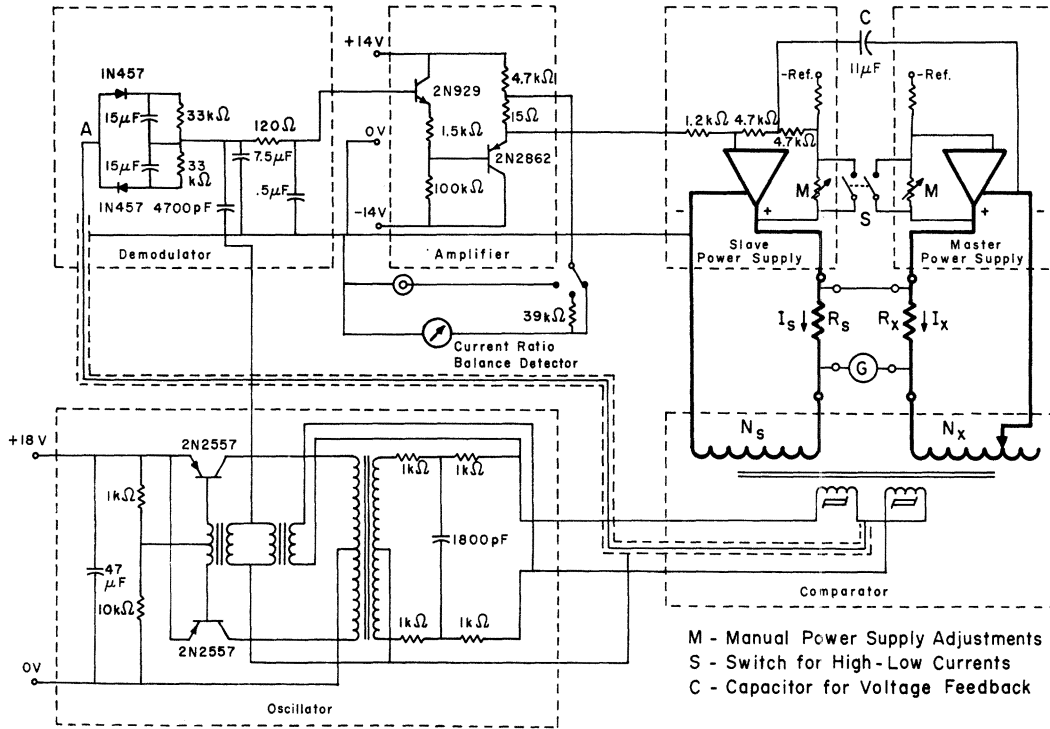


Fig. 4. Circuit diagram of magnetic modulator.

By definition

$$R_s = R_{sN}(1 + \alpha_s)$$

$$R_x = R_{xN}(1 + \alpha_x).$$

At balance

$$N_s = 1000(1 + \epsilon_s - \epsilon_x)$$

$$R_x = N_x \frac{R_s}{N_s}.$$

Since

$$\frac{N_x}{1000} = \frac{R_{xN}}{R_{sN}} = r_n,$$

by substituting

$$R_{xN}(1 + \alpha_x) = 1000 r_n \frac{R_{sN}(1 + \alpha_s)}{1000(1 + \epsilon_s - \epsilon_x)}$$

$$r_n(1 + \alpha_x) = r_n \frac{1 + \alpha_s}{1 + \epsilon_s - \epsilon_x}$$

$$1 + \alpha_x = \frac{1 + \alpha_s}{1 + \epsilon_s - \epsilon_x},$$

and since $\epsilon_s = \alpha_s$

$$1 + \alpha_x = \frac{1 + \epsilon_s}{1 + (\epsilon_s - \epsilon_x)} \cong (1 + \epsilon_s)(1 - \epsilon_s + \epsilon_x)$$

$$1 + \alpha_x \cong 1 + \epsilon_s - \epsilon_s + \epsilon_x - \epsilon_s(\epsilon_s - \epsilon_x)$$

$$\alpha_x \cong \epsilon_x.$$

The approximation used in the above equations assumes that both $(\epsilon_s - \epsilon_x)^2$ and $\epsilon_s(\epsilon_s - \epsilon_x)$ are negligibly small.

B. Magnetic Modulator

The magnetic modulator is a most important component of the bridge. To provide adequate resolution at low operating currents it should have high sensitivity. One of the purposes in building the bridge was to determine the maximum sensitivity which could be realized. High sensitivity is however not a guarantee of accuracy. To obtain the required accuracy, not only must the sensitivity be adequate, but also the modulator must be insensitive to such variables as temperature, ambient magnetic fields and alternating currents in the ratio winding, either internally or externally generated.

Another design requirement which should be met is that the current ratio balance detector is not affected by the manual setting of the slave power supply.

Constructional details of the modulator cores and windings are given in the Appendix. The circuit diagram is given in Fig. 4. In addition to the comparator, the modulator has the following main components: a 700 Hz square wave oscillator, a demodulator, and a direct current amplifier. The modulator signal at A is a peaked alternating voltage of mainly fundamental frequency. Direct current in the ratio windings adds even harmonics to this signal, increasing the amplitude of the peaks of one polarity and decreasing those of opposite polarity. The demodulator generates a direct voltage proportional to the difference between the amplitudes of the positive and negative peaks, and is thus proportional to the net direct current in the ratio windings. This signal is then amplified in a transistor amplifier before it is applied to the current ratio balance detector and to the feedback connection of the slave power supply.

MODULATOR PERFORMANCE AND LIMITATIONS

A. Zero Stability and Sensitivity

The highest usable sensitivity of the magnetic modulator is limited by its zero stability, which is affected by such uncontrolled parameters as the temperature of the modulator cores and the amplitude of the output of the driving oscillator. These parameters can be stabilized by appropriate controls, but a more satisfactory solution is direct feedback from the modulator cores to the square wave oscillator. Even though the saturation flux density of a magnetic core is temperature dependent the modulator cores may be driven to the same degree of saturation independent of their temperature by a feedback signal, generated by core saturation, which controls the switching point of the square wave oscillator. This modification to the Royer-type square wave oscillator improves the zero stability of the modulator significantly.

Figure 5 is a recording of the modulator output after a warm-up of about 15 minutes. The short-time zero stability is about two or three micro-ampere-turns, the long-term zero stability after warm-up is about $10 \mu\text{AT}$.

B. Memory Effect

When the modulator is subjected, even momentarily, to a large current unbalance a semi-permanent zero shift occurs. This shift can be reduced by circuit design but never completely eliminated. Previous investigators [14] have found that driving the modulator cores into saturation is beneficial and the bridge modulator is designed to do this. The memory zero shift due to a current unbalance of one ampere-turn is still significant however and a balancing procedure has been adopted which avoids such large current unbalances.

C. Ac Rejection

Ideally, the magnetic modulator should be insensitive to alternating components of the ratio winding currents. This characteristic can be approached by circuit design but never completely realized. However, a procedure for balancing the bridge can be adopted which will reduce their effect on the accuracy of the measurement to below the $10 \mu\text{AT}$ drift level of the modulator.

Alternating currents in the ratio windings are either of internal or external origin. Internally generated alternating currents are those induced in the ratio winding by modulator action. The magnitude of the zero shift produced by these currents can be studied experimentally by changing the resistance in a ratio winding and observing the corresponding zero shift of the detector. Figure 6 illustrates this characteristic of the bridge modulator-detector. It will be noticed that practically no zero shift is produced as the resistance load is varied from zero to 1000 ohms. Above this load however a significant zero shift occurs.

To realize the low zero shift illustrated in Fig. 6 a high permeability magnetic shield is essential but in addition considerable care has to be taken in choosing the operat-

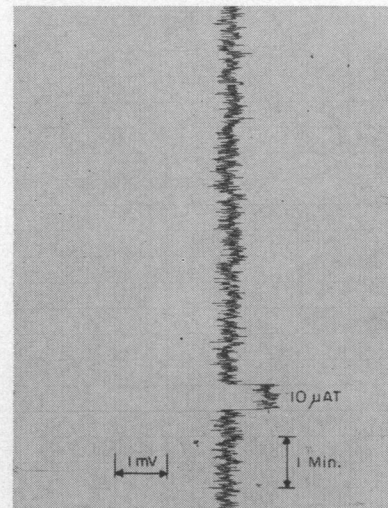


Fig. 5. Noise and drift of current ratio balance detector.

ing point of the modulator-detector. Because of core mismatch zero current in the ratio windings does not necessarily mean zero second harmonic component in the modulator output. Similarly, since the detector circuit includes a dc amplifier which may have a zero offset of its own, an indicated zero does not necessarily mean a zero second harmonic component in the modulator output. To obtain the proper operating point of the detector, the effect of core mismatch is reduced by the adjustment of the current in a bias winding until on second harmonic component remains in the modulator output. A bias in the dc amplifier is then set to make the detector read zero.

The effect of this zero shift on the accuracy of the measurement can be eliminated by keeping the resistance connected to the ratio windings constant. For this purpose the use of power supplies whose output impedance is independent of the output voltage setting is highly desirable. Opening of the circuit of a ratio winding in order to realize zero current is definitely not a proper operating procedure.

Externally generated alternating currents in the ratio windings are those produced by the ripple content of the master and slave power supplies. Figure 7 illustrates the magnitude of the alternating current of different frequencies in the master power supply which will produce a $10 \mu\text{AT}$ zero shift. This magnitude depends to a great extent on the value of the standard resistor connected to the 1000-turn N_s winding. When this resistance is low, the effective alternating current seen by the modulator is greatly reduced by current transformer action.

For low current level operation the master power supply can be a well filtered, regulated supply, with negligibly small ripple content. For high current operation (100 A) the master power supply could be an unfiltered three-phase rectifier having a large 180 Hz ripple content. If the standard resistance is 10 ohms, the curve on Fig. 7 shows that 10 AT of 180 Hz ripple current are

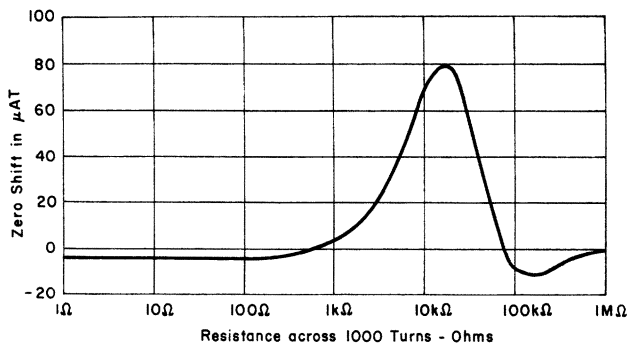


Fig. 6. Zero shift produced by resistance variation.

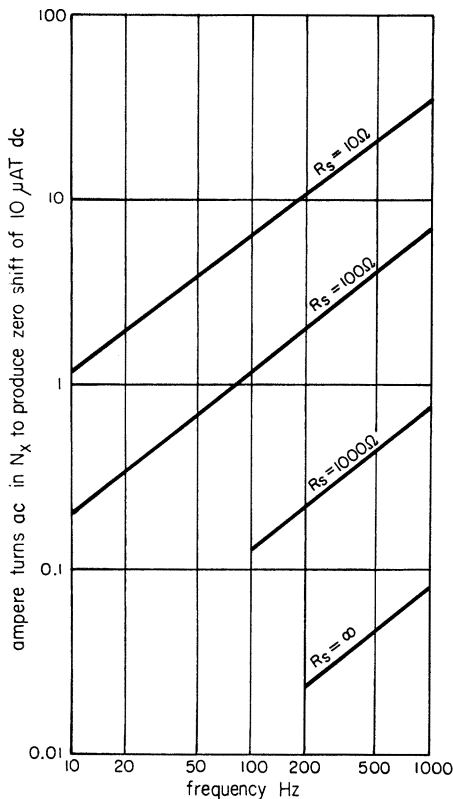


Fig. 7. Tolerable level of ac currents generated in master power supply.

required to produce a zero shift of $10 \mu\text{AT}$ dc. This alternating current is a 100 percent ripple on 14 AT of direct current, so that the corresponding zero shift is less than one ppm. At $R_s = 100$ ohms a 20 percent ripple content would produce the same zero shift.

The effect of ac ripple in the slave power supply should be considered with care. Ripple in this power supply cannot completely be avoided since the modulator output contains a large ripple content. Hence when the feedback loop is closed alternating current will flow in the corresponding ratio winding. The magnitude of this alternating current is reduced by a suitable filter, by a high permeability shield, and by a suitable high-frequency roll off in the coupling network to the slave power supply. A particular circuit design can be tested from this point of view by measuring the zero shift produced by closing the feedback loop with the N_x turns

open circuited. This measurement gives the zero shift produced by a loop gain variation from zero to maximum. If this shift is sufficiently small, one is assured that any zero shift produced by relatively small loop gain variations which may occur will not affect the measurement.

BALANCING PROCEDURE

To avoid errors caused by thermal voltages in the potential circuit it is customary to reverse the current flow in the resistors, leaving the potential circuit unaltered. To adopt this procedure in the current comparator bridge the current would have to be reversed in both resistors. This would open the feedback loop momentarily and large current unbalances could occur which would give rise to a memory zero shift leading to an error in the measurement. A suitable operating procedure is to vary the current by switching the control circuit of the power supplies. In this way the feedback loop is never opened and comparator memory errors can be eliminated. Most commercially available power supplies are however nonreversible so that the maximum variation of current realizable is from zero to maximum. The correct bridge balance is obtained if both balances are unaffected when the current is switched from zero to maximum. The voltage feedback produced by capacitor C in Fig. 4 has been found very effective in reducing switching surges.

There are two disadvantages to this balancing procedure. The first is a reduced voltage balance sensitivity. The second is apparent only when one attempts to measure resistances at high currents where their values are current dependent. The high current balance then takes several minutes to stabilize but finally reaches an equilibrium condition when thermal balance is achieved. The low current balance corresponding to this temperature condition can only be measured by taking a quick balance after switching. This is not a satisfactory method and to avoid it the use of reversing dc power supplies, which have recently become commercially available, is being considered.

ACCURACY OF BRIDGE MEASUREMENTS

The accuracy of a current comparator as a current ratio indicator can quite readily be made better than one ppm particularly if a high permeability magnetic shield is used. The error of the bridge comparator on the 1000-to-1000-turns ratio was found to be 5×10^{-9} . This high accuracy applies however only to an integral number of turns. The fractional turn errors, which depend on a resistive divider network, have to be considered separately.

The resistive divider network is shown in Fig. 3. The network is adjusted so that the 50 ohm, 500 ohm, and 5000 ohm resistors carry 0.1, 0.01, and 0.001 of the current while the resistor R' carries the remaining 0.889. The decimal fractional current can be passed through an integral number of turns from -1 to $+10$. This arrange-

ment provides a self-checking feature for the divider network. The resistive divider networks of the bridge should be adjusted to better than 0.1 of one step of the last dial. To be able to keep this setting the following precautions are necessary:

- 1) The switches in series with the 50 ohm resistor must be of high quality so that any variation of their contact resistance remains less than 5 milliohms.
- 2) The resistors, particularly R' and the 50 ohm, should have the same temperature coefficient and should be mounted in an adequate heat sink to equalize their temperatures. An aluminum block was found adequate for the purpose.
- 3) The resistance of the ten-turn winding, and of the leads which connect to the 50 ohms, should be small and equalized.

To realize the full accuracy capabilities of the current comparator when measuring resistances on the bridge it is essential that the proper balancing procedure be followed. The direct reading accuracy of the bridge is then as good as its resolution.

For low resistance measurements (milliohm range) the resolution of the bridge reading is limited by the sensitivity and zero stability of the voltage balance detector. The voltage sensitivity of this detector galvanometer is high because the external circuit resistance, consisting of the resistors being compared, is low. For high resistance measurements (above 100 ohms) the resolution of the bridge is limited by the sensitivity and zero stability of the current ratio balance detector (magnetic modulator). The zero stability of the latter is 10 μ AT so that a minimum operating level of 10 AT is required to obtain one ppm resolution. This corresponds to 10 mA in 1000 turns. At lower operating currents the resolution of the bridge is reduced in proportion.

OPERATING RANGE

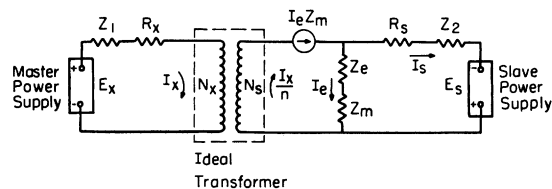
The lower current limit of the bridge is 10 mA on the standard side if ppm resolution is to be achieved. The upper current limit is set by the current carrying capacity of the windings. The upper resistance limit is dictated by feedback considerations and is about 1000 ohms. As the resistance on the standard side increases the gain of the feedback loop is reduced and eventually a point is reached where the stabilizing action of the loop is not sufficient to produce a stable current ratio balance. This can be analyzed with the aid of the equivalent circuit [13] of the self-balancing direct current comparator shown in Fig. 8. This equivalent circuit is similar to that of a current transformer having R_s as the burden but in which the core magnetizing impedance Z_e has been increased by the equivalent modulator impedance Z_m . The effective modulator impedance $Z_m = SN_sG$ where

- S is the modulator sensitivity, 80 V/AT at zero frequency,
- N_s is about 1000 turns, and

G is the voltage gain of the electronic amplifier measured between the modulator output and the output of the slave power supply. This gain varies with the power supply setting but is never lower than seven.

Both impedances Z_e and Z_m vary with frequency as indicated in Fig. 9. The frequency response of Z_m is determined almost entirely by the filter in the demodulator.

As the burden of a current transformer is increased its accuracy is decreased; the self-balancing current comparator has the same characteristic. As the value of R_s is increased to 1000 ohms its current ratio stabilizing action is reduced to the minimum tolerable level. This establishes the upper resistance limit of the bridge. This limit is not of fundamental but of a practical nature.



- n = turns ratio = N_s/N_x
- Z_1 & Z_2 = leakage impedances
- R_x = unknown resistance
- R_s = standard resistance
- Z_e = magnetizing impedance
- Z_m = equivalent modulator impedance
- I_e = error current
- E_x & E_s = power supply voltages

Fig. 8. Equivalent circuit of self-balancing current comparator.

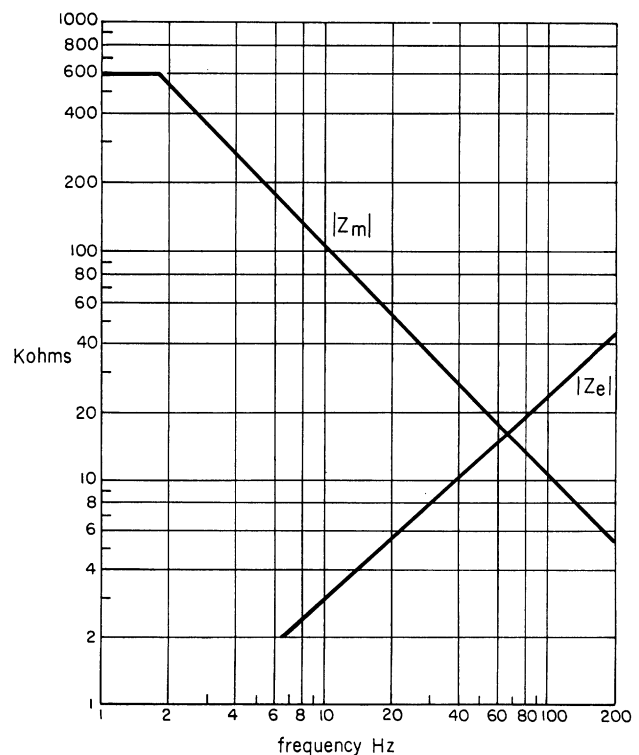


Fig. 9. $|Z_m$ and $|Z_e|$ for $N_s=1000$ turns.

ALTERNATIVE BRIDGE CONNECTION

The above mentioned upper resistance limit of the bridge exists only for four-terminal measurements. This limitation can be nearly completely removed if two-terminal measurements are permissible. An alternative bridge connection, illustrated in Fig. 10, can then be used. In this operating mode the currents through both resistors are derived from the master power supply. The slave power supply still keeps the comparator balanced, but is required to supply only the small difference in potential drops between N_s and N_x . The capacitive voltage feedback network used in four-terminal measurements should not be used in this bridge connection as the voltage variations of the two supplies are not the same when the current level is switched.

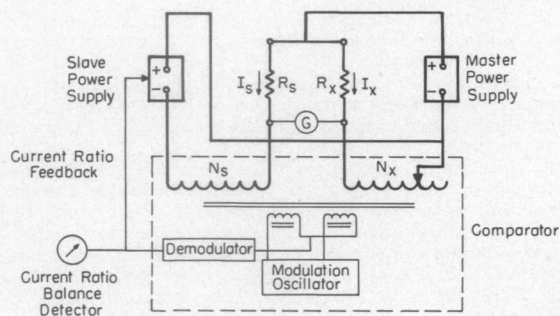


Fig. 10. Alternative connection of the direct-current-comparator ratio bridge.

APPLICATIONS

Applications of the bridge include:

- 1) scaling of standard resistances, particularly below 100 ohms,
- 2) measurement of the load coefficient of low value standards, and
- 3) calibration of shunts at full rated current.

The bridge has been used at the National Research Council for the calibration of standard resistances from 1000 ohms to 100 micro-ohms, at ratios up to 1000/1 in a single step. The results have been cross-checked, and also compared with those made on conventional bridges. The agreement appears to justify confidence in the accuracy claims for the comparator bridge [15].

CONCLUSIONS

It has been shown that the advantages of ratio transformers can be used in direct current resistance measurements. However, the modulation technique required for this purpose introduces a noise level in the current ratio balance detector which is not present in ac ratio transformers. With careful design it has been possible to reduce this noise level to less than $10 \mu\text{AT}$.

A bridge has been developed for the measurement of four-terminal resistances which can be used either as a full range direct reading resistance bridge or as a limited range direct reading deviation bridge in which the nom-

inal resistance ratio can be set, to three significant digits, at any integral value up to 1000 to 1.

Bridge balance consists of two simultaneous, rather than consecutive, balances. Any instability which is large enough to affect the result is thus immediately indicated, and it limits the resolution which can be attained in a measurement. A maximum resolution of one ppm has been incorporated in the bridge. When the recommended operating procedure is followed the bridge has a direct reading accuracy as good as the resolution of the measurement.

Bridge resolution is not always limited by the zero stability of the modulator. It has been found that in the measurement of fractional ohm resistances the noise level or zero stability of the potential circuit is often the limiting factor.

Figure 11 is an illustration of the bridge. Its application in standardizing laboratories is indicated. Of particular importance is the fact that its operating mode permits the calibration of low value shunts at full operating current with an accuracy limited only by their noise level and stability.

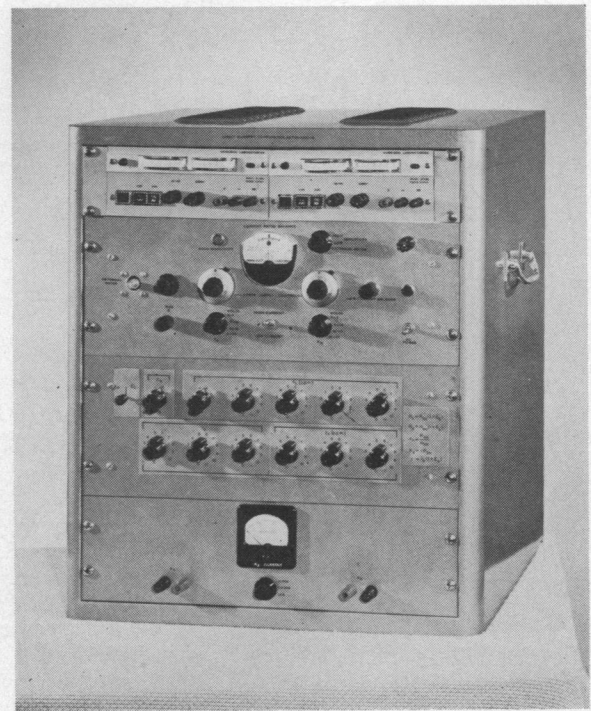


Fig. 11. Front view of direct-current-comparator ratio bridge.

APPENDIX

CONSTRUCTION OF THE CURRENT COMPARATOR

- 1) Magnetic cores—5.0 in ID by 5.5 inches OD by 0.5 inch height, 0.002 inch HyMu 80, aluminum cases.
- 2) Modulation-detection winding—single layer of No. 24 Formex, 1006 turns on each core.
- 3) Electrostatic screens—copper foil.
- 4) Magnetic shield—annealed 0.015 inch Mumetal,

surrounding both cores.

inner toroid—10 layers, 3.5 inches height.

outer toroid—10 layers, 3.5 inches height.

end toroids—4.5 inches ID by 6.0 inches OD by 0.75 inch height.

5) Outer shield—3.625 inches ID by 6.75 inches OD by 3.94 inches height, 0.0625 inch copper.

6) Ratio windings—Formex wire

	Current rating
N_s —1000 turns, No. 18	1 A
11 turns, No. 18, tapped	1 A
N_x —1000 turns, No. 18, tapped	1 A
110 turns, No. 18, tapped	1 A
11 turns, No. 18, tapped	1 A
100 turns, 2 in parallel No. 18	4 A
10 turns, No. 10	20 A
1 turn, 10 in parallel No. 10	150 A
ϵ_s —11 turns, No. 10, tapped	—
ϵ_x —11 turns, No. 10, tapped	—

Note: Insulated gaps are provided in all shields to prevent the formation of a short-circuiting turn.

Current rating of ϵ_s and ϵ_x limited by the resistive divider (1 A).

REFERENCES

- [1] A. M. Thompson, "The precise measurement of small capacitances," *IRE Trans. on Instrumentation*, vol. I-7, pp. 245-253, December 1958.
- [2] M. C. McGregor, J. F. Hersh, R. D. Cutkosky, F. K. Harris, and F. R. Kotter, "New apparatus at the National Bureau of Standards for absolute capacitance measurements," *IRE Trans. on Instrumentation*, vol. 17, pp. 253-261, December 1958.
- [3] D. L. H. Gibbings, "An alternating-current analogue of the Kelvin Double Bridge," *J. IEE (London)*, vol. 109, Pt. C., pp. 307-316, September 1962.
- [4] J. J. Hill and A. P. Miller, "An a.c. double bridge with inductively coupled ratio arms for precision platinum-resistance thermometry," *Proc. IEE (London)*, vol. 110, pp. 453-458, February 1963.
- [5] N. L. Kusters and O. Petersons, "A transformer-ratio-arm bridge for high-voltage capacitance measurements," *IEEE Trans. on Communication and Electronics*, vol. 82, pp. 606-611, November 1963.
- [6] A. M. Thompson, "AC bridge methods for the measurement of three-terminal admittances," *IEEE Trans. on Instrumentation and Measurement*, vol. IM-13, pp. 189-197, December 1964.
- [7] R. D. Cutkosky, "Active and passive direct-reading ratio sets for the comparison of audio-frequency admittances," *IEEE Trans. on Instrumentation and Measurement*, vol. IM-13, pp. 243-250, December 1964.
- [8] J. J. Hill, "The calibration of dc resistance standards by an ac method," *IEEE Trans. on Instrumentation and Measurement*, vol. IM-13, pp. 239-243, December 1964.
- [9] T. M. Dauphinee, "An isolating potential comparator," *Canad. J. Phys.*, vol. 31, pp. 577-591, May 1953.
- [10] N. L. Kusters, "The precise measurement of current ratios," *IEEE Trans. on Instrumentation and Measurement*, vol. IM-13, pp. 197-209, December 1964.
- [11] N. L. Kusters, W. J. M. Moore and P. N. Miljanic, "A current comparator for the precision measurement of d-c ratios," *IEEE Trans. on Communication and Electronics*, vol. 83, pp. 22-27, January 1964.
- [12] S. K. Basu and N. L. Kusters, "Comparison of standard resistors by the dc current-comparator," *IEEE Trans. on Instrumentation and Measurement*, vol. IM-14, pp. 149-156, September 1965.
- [13] M. P. MacMartin and N. L. Kusters, "A self-balancing direct current comparator for 20 000 amperes," *IEEE Trans. on Magnetics*, vol. MAG-1, pp. 396-402, December 1965.
- [14] F. C. Williams and S. W. Noble, "The fundamental limitations of the second harmonic type of magnetic modulator as applied to the amplification of small dc signals," *J. IEE (London)*, vol. 97, Pt. 2, pp. 445-459, August 1950.
- [15] A. F. Dunn, "Increased accuracy for resistance measurements," this issue, page 220.

Increased Accuracy for Resistance Measurements

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Abstract—The effect of power dissipation on the performance of four-terminal standard resistors has been investigated, using the direct-current-comparator ratio bridge. A theoretical analysis of a typical standard resistor has also been made, which indicates that the average temperature increase of the resistance wire in an oil-filled resistor is approximated by

$$T - T_0 = RI^2[6(1 - e^{-0.001I}) + 1]$$

and that the Peltier effect at the copper-manganin junctions may contribute errors of 1 to 10 parts per million of resistance for the lower-valued resistors. Recommendations are made for design and construction changes in standard resistors, which will improve the accuracy with which they may be used.

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INTRODUCTION

IMPROVED PRECISION in the measurement of resistance standards may be obtained by increasing the current through the resistors and by improving the sensitivity of the detector.

The direct-current-comparator ratio bridge [1] has introduced a new aspect to the comparison of standard resistors by measuring the current ratios corresponding to voltage equality across the resistors, as in the comparison circuit in Fig. 1, in which at balance

$$\frac{XI_X}{SI_S} = 1 \quad \text{or} \quad \frac{X}{S} = \frac{I_S}{I_X} = \frac{N_X(1 + \Delta_N)}{N_S} \quad (1)$$

where N_x and N_s are integral numbers of turns on the