Resistor ratio dividers based on N equal resistors

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Resistor network series such as LT5400, TDP1603, NOMCA1603, TOMC1603, ORN and many more are known to have good t.c. tracking and some of them also have quite low 1/f noise [1], [2].

On the other hand dividers are needed when it comes to voltage reference circuits such as LTZ1000 for either setting the operating temperature of the internal oven or scaling of the output voltage.
In [3] a ratio of 13 k : 1 k is shown to set the oven temperature to 60°C. Alltough the divider is given with exact numbers for the resistors only the ratio is important and fractions or multiple of that resistors forming the same ratio would lead to the same results.
By now, most of the circuits presented used either precision wirewound resistors or fixed ratio dividers such as VHD from Vishay in hermetically sealed package, with ratios such as 13k:1k or similar. This dividers are with no doubt superior in either t.c. tracking and noise, though expensive and the leadtime of several months is quite long. Even custom ratios are possible. Nevertheless, getting a complete and trimmed 10 V voltage reference based on such custom parts up and running can take almost forever and is thus not practical.
The aformentioned networks are cheap and easy to score from several distributors. Though, only a few engineers are aware of what is possible with only N equal resistors.
In [4] an equivalent resistance made by N equal resistors is discussed. Based on the results presented it can be concluded, that resistive dividers can be made out of combinations of serial, parallel, bridge and cascade connections of N equal resistors, too. Since such combinations also allow for different taps within the same resistor arrangement even and odd divider ratios can be created to whatever is needed.
The most common approach is to arrange the resistors in a fashion, that the resulting network matches the absolute values as given e.g. per datasheet of the LTZ reference circuit. As already discussed this is not needed, as mostly the ratio is important.
In 1 some of the possible ratios already found with up to 8 equal resistors are presented. The results found use cascades as well as serial and parallel connections in a limited extend only. Thus, the table is far away from being complete and having an algorithm available to find all possible combinations is one of the current tasks.

Table 1

								Voltage ratio	Resistance ratio
-	-	-	-	-	-	-	-	1:1	
-	-	-	-	-	-	-	|	8:1	7:1
-	-	-	-	-	-	|	-	7:1	6:1
-	-	-	-	-	-	|	|	13:1	12:1
-	-	-	-	-	|	-	-	6:1	5:1
-	-	-	-	-	|	-	|	17:1	16:1
-	-	-	-	-	|	|	-	11:1	10:1
-	-	-	-	-	|	|	|	16:1	15:1
-	-	-	-	|	-	-	-	5:1	4:1
-	-	-	-	|	-	-	|	19:1	18:1
-	-	-	-	|	-	|	-	14:1	13:1
-	-	-	-	|	-	|	|	23:1	22:1
-	-	-	-	|	|	-	-	9:1	8:1
-	-	-	-	|	|	-	|	22:1	21:1
-	-	-	-	|	|	|	-	13:1	12:1
-	-	-	-	|	|	|	|	17:1	16:1
-	-	-	|	-	-	-	-	4:1	3:1
-	-	-	|	-	-	-	|	19:1	18:1
-	-	-	|	-	-	|	-	15:1	14:1
-	-	-	|	-	-	|	|	26:1	25:1
-	-	-	|	-	|	-	-	11:1	10:1
-	-	-	|	-	|	-	|	29:1	28:1
-	-	-	|	-	|	|	-	18:1	17:1
-	-	-	|	-	|	|	|	25:1	24:1
-	-	-	|	|	-	-	-	7:1	6:1
-	-	-	|	|	-	-	|	24:1	23:1
-	-	-	|	|	-	|	-	17:1	16:1
-	-	-	|	|	-	|	|	27:1	26:1
-	-	-	|	|	|	-	-	10:1	9:1
-	-	-	|	|	|	-	|	23:1	22:1
-	-	-	|	|	|	|	-	13:1	12:1
-	-	-	|	|	|	|	|	16:1	15:1
-	-	|	-	-	-	-	-	3:1	2:1
-	-	|	-	-	-	-	|	17:1	16:1
-	-	|	-	-	-	|	-	14:1	13:1
-	-	|	-	-	-	|	|	25:1	24:1
-	-	|	-	-	|	-	-	11:1	10:1
-	-	|	-	-	|	-	|	30:1	29:1
-	-	|	-	-	|	|	-	19:1	18:1
-	-	|	-	-	|	|	|	27:1	26:1
-	-	|	-	|	-	-	-	8:1	7:1
-	-	|	-	|	-	-	|	29:1	28:1
-	-	|	-	|	-	|	-	21:1	20:1
-	-	|	-	|	-	|	|	34:1	33:1
-	-	|	-	|	|	-	-	13:1	12:1
-	-	|	-	|	|	-	|	31:1	30:1
-	-	|	-	|	|	|	-	18:1	17:1
-	-	|	-	|	|	|	|	23:1	22:1
-	-	|	|	-	-	-	-	5:1	4:1
-	-	|	|	-	-	-	|	22:1	21:1
-	-	|	|	-	-	|	-	17:1	16:1
-	-	|	|	-	-	|	|	29:1	28:1
-	-	|	|	-	|	-	-	12:1	11:1
-	-	|	|	-	|	-	|	31:1	30:1
-	-	|	|	-	|	|	-	19:1	18:1
-	-	|	|	-	|	|	|	26:1	25:1
-	-	|	|	|	-	-	-	7:1	6:1
-	-	|	|	|	-	-	|	23:1	22:1
-	-	|	|	|	-	|	-	16:1	15:1
-	-	|	|	|	-	|	|	25:1	24:1
-	-	|	|	|	|	-	-	9:1	8:1
-	-	|	|	|	|	-	|	20:1	19:1
-	-	|	|	|	|	|	-	11:1	10:1
-	-	|	|	|	|	|	|	13:1	12:1
-	|	-	-	-	-	-	-	2:1	1:1
-	|	-	-	-	-	-	|	13:1	12:1
-	|	-	-	-	-	|	-	11:1	10:1
-	|	-	-	-	-	|	|	20:1	19:1
-	|	-	-	-	|	-	-	9:1	8:1
-	|	-	-	-	|	-	|	25:1	24:1
-	|	-	-	-	|	|	-	16:1	15:1
-	|	-	-	-	|	|	|	23:1	22:2
-	|	-	-	|	-	-	-	7:1	6:1
-	|	-	-	|	-	-	|	26:1	25:1

								Voltage ratio	Resistance ratio
-	|	-	-	|	-	|	-	19:1	18:1
-	|	-	-	|	-	|	|	31:1	30:1
-	|	-	-	|	|	-	-	12:1	11:1
-	|	-	-	|	|	-	|	29:1	28:1
-	|	-	-	|	|	|	-	17:1	16:1
-	|	-	-	|	|	|	|	22:1	21:1
-	|	-	|	-	-	-	-	5:1	4:1
-	|	-	|	-	-	-	|	23:1	22:1
-	|	-	|	-	-	|	-	18:1	18:1
-	|	-	|	-	-	|	|	31:1	30:1
-	|	-	|	-	|	-	-	13:1	12:1
-	|	-	|	-	|	-	|	34:1	33:1
-	|	-	|	-	|	|	-	21:1	20:1
-	|	-	|	-	|	|	|	29:1	28:1
-	|	-	|	|	-	-	-	8:1	7:1
-	|	-	|	|	-	-	|	27:1	26:1
-	|	-	|	|	-	|	-	19:1	18:1
-	|	-	|	|	-	|	|	30:1	29:1
-	|	-	|	|	|	-	-	11:1	10:1
-	|	-	|	|	|	-	|	25:1	24:1
-	|	-	|	|	|	|	-	14:1	13:1
-	|	-	|	|	|	|	|	17:1	16:1
-	|	|	-	-	-	-	-	3:1	2:1
-	|	|	-	-	-	-	|	16:1	15:1
-	|	|	-	-	-	|	-	13:1	12:1
-	|	|	-	-	-	|	|	23:1	22:1
-	|	|	-	-	|	-	-	10:1	9:1
-	|	|	-	-	|	-	|	27:1	26:1
-	|	|	-	-	|	|	-	17:1	16:1
-	|	|	-	-	|	|	|	24:1	23:1
-	|	|	-	|	-	-	-	7:1	6:1
-	|	|	-	|	-	-	|	25:1	24:1
-	|	|	-	|	-	|	-	18:1	17:1
-	|	|	-	|	-	|	|	29:1	28:1
-	|	|	-	|	|	-	-	11:1	10:1
-	|	|	-	|	|	-	|	26:1	25:1
-	|	|	-	|	|	|	-	15:1	14:1
-	|	|	-	|	|	|	|	19:1	18:1
-	|	|	|	-	-	-	-	4:1	3:1
-	|	|	|	-	-	-	|	17:1	16:1
-	|	|	|	-	-	|	-	13:1	12:1
-	|	|	|	-	-	|	|	22:1	21:1
-	|	|	|	-	|	-	-	9:1	8:1
-	|	|	|	-	|	-	|	23:1	22:1
-	|	|	|	-	|	|	-	14:1	13:1
-	|	|	|	-	|	|	|	19:1	18:1
-	|	|	|	|	-	-	-	5:1	4:1
-	|	|	|	|	-	-	|	16:1	15:1
-	|	|	|	|	-	|	-	11:1	10:1
-	|	|	|	|	-	|	|	17:1	16:1
-	|	|	|	|	|	-	-	6:1	5:1
-	|	|	|	|	|	-	|	13:1	12:1
-	|	|	|	|	|	|	-	7:1	6:1
-	|	|	|	|	|	|	|	8:1	7:1
|	-	-	-	-	-	-	-	1:1	
|	-	-	-	-	-	-	|	7:1	6:1
|	-	-	-	-	-	|	-	6:1	5:1
|	-	-	-	-	-	|	|	11:1	10:1
|	-	-	-	-	|	-	-	5:1	4:1
|	-	-	-	-	|	-	|	14:1	13:1
|	-	-	-	-	|	|	-	9:1	8:1
|	-	-	-	-	|	|	|	13:1	12:1
|	-	-	-	|	-	-	-	4:1	3:1
|	-	-	-	|	-	-	|	15:1	14:1
|	-	-	-	|	-	|	-	11:1	10:1
|	-	-	-	|	-	|	|	18:1	17:1
|	-	-	-	|	|	-	-	7:1	6:1
|	-	-	-	|	|	-	|	17:1	16:1
|	-	-	-	|	|	|	-	10:1	9:1
|	-	-	-	|	|	|	|	13:1	12:1
|	-	-	|	-	-	-	-	3:1	2:1
|	-	-	|	-	-	-	|	14:1	13:1
|	-	-	|	-	-	|	-	11:1	10:1
|	-	-	|	-	-	|	|	19:1	18:1

								Voltage ratio	Resistance ratio
|	-	-	|	-	|	-	-	8:1	7:1
|	-	-	|	-	|	-	|	21:1	20:1
|	-	-	|	-	|	|	-	13:1	12:1
|	-	-	|	-	|	|	|	18:1	17:1
|	-	-	|	|	-	-	-	5:1	4:1
|	-	-	|	|	-	-	|	17:1	16:1
|	-	-	|	|	-	|	-	12:1	11:1
|	-	-	|	|	-	|	|	19:1	18:1
|	-	-	|	|	|	-	-	7:1	6:1
|	-	-	|	|	|	-	|	16:1	15:1
|	-	-	|	|	|	|	-	9:1	8:1
|	-	-	|	|	|	|	|	11:1	10:1
|	-	|	-	-	-	-	-	2:1	1:1
|	-	|	-	-	-	-	|	11:1	10:1
|	-	|	-	-	-	|	-	9:1	8:1
|	-	|	-	-	-	|	|	16:1	15:1
|	-	|	-	-	|	-	-	7:1	6:1
|	-	|	-	-	|	-	|	19:1	18:1
|	-	|	-	-	|	|	-	12:1	11:1
|	-	|	-	-	|	|	|	17:1	16:1
|	-	|	-	|	-	-	-	5:1	4:1
|	-	|	-	|	-	-	|	18:1	17:1
|	-	|	-	|	-	|	-	13:1	12:1
|	-	|	-	|	-	|	|	21:1	20:1
|	-	|	-	|	|	-	-	8:1	7:1
|	-	|	-	|	|	-	|	19:1	18:1
|	-	|	-	|	|	|	-	11:1	10:1
|	-	|	-	|	|	|	|	14:1	13:1
|	-	|	|	-	-	-	-	3:1	2:1
|	-	|	|	-	-	-	|	13:1	12:1
|	-	|	|	-	-	|	-	10:1	9:1
|	-	|	|	-	-	|	|	17:1	16:1
|	-	|	|	-	|	-	-	7:1	6:1
|	-	|	|	-	|	-	|	18:1	17:1
|	-	|	|	-	|	|	-	11:1	10:1
|	-	|	|	-	|	|	|	15:1	14:1
|	-	|	|	|	-	-	-	4:1	3:1
|	-	|	|	|	-	-	|	13:1	12:1
|	-	|	|	|	-	|	-	9:1	8:1
|	-	|	|	|	-	|	|	14:1	13:1
|	-	|	|	|	|	-	-	5:1	4:1
|	-	|	|	|	|	-	|	11:1	10:1
|	-	|	|	|	|	|	-	6:1	5:1
|	-	|	|	|	|	|	|	7:1	6:1
|	|	-	-	-	-	-	-	1:1	
|	|	-	-	-	-	-	|	6:1	5:1
|	|	-	-	-	-	|	-	5:1	4:1
|	|	-	-	-	-	|	|	9:1	8:1
|	|	-	-	-	|	-	-	4:1	3:1
|	|	-	-	-	|	-	|	11:1	10:1
|	|	-	-	-	|	|	-	7:1	6:1
|	|	-	-	-	|	|	|	10:1	9:1
|	|	-	-	|	-	-	-	3:1	2:1
|	|	-	-	|	-	-	|	11:1	10:1
|	|	-	-	|	-	|	-	8:1	7:1
|	|	-	-	|	-	|	|	13:1	12:1
|	|	-	-	|	|	-	-	5:1	4:1
|	|	-	-	|	|	-	|	12:1	11:1
|	|	-	-	|	|	|	-	7:1	6:1
|	|	-	-	|	|	|	|	9:1	8:1
|	|	-	|	-	-	-	-	2:1	1:1
|	|	-	|	-	-	-	|	9:1	8:1
|	|	-	|	-	-	|	-	7:1	7:1
|	|	-	|	-	-	|	|	12:1	11:1
|	|	-	|	-	|	-	-	5:1	4:1
|	|	-	|	-	|	-	|	13:1	12:1
|	|	-	|	-	|	|	-	8:1	7:1
|	|	-	|	-	|	|	|	11:1	10:1
|	|	-	|	|	-	-	-	3:1	2:1
|	|	-	|	|	-	-	|	10:1	9:1
|	|	-	|	|	-	|	-	7:1	6:1
|	|	-	|	|	-	|	|	11:1	10:1
|	|	-	|	|	|	-	-	4:1	3:1
|	|	-	|	|	|	-	|	9:1	8:1

								Voltage ratio	Resistance ratio
|	|	-	|	|	|	|	-	5:1	4:1
|	|	-	|	|	|	|	6:1	5:1
|	|	|	-	-	-	-	-	1:1	
|	|	|	-	-	-	-	|	5:1	4:1
|	|	|	-	-	-	|	-	4:1	3:1
|	|	|	-	-	-	|	|	7:1	6:1
|	|	|	-	-	|	-	-	3:1	2:1
|	|	|	-	-	|	-	|	8:1	7:1
|	|	|	-	-	|	|	-	5:1	4:1
|	|	|	-	-	|	|	|	7:1	6:1
|	|	|	-	|	-	-	-	2:1	1:1
|	|	|	-	|	-	-	|	7:1	6:1
|	|	|	-	|	-	|	-	5:1	4:1
|	|	|	-	|	-	|	|	8:1	7:1
|	|	|	-	|	|	-	-	3:1	2:1
|	|	|	-	|	|	-	|	7:1	6:1
|	|	|	-	|	|	|	-	4:1	3:1
|	|	|	-	|	|	|	|	5:1	4:1
|	|	|	|	-	-	-	-	1:1	
|	|	|	|	-	-	-	|	4:1	3:1
|	|	|	|	-	-	|	-	3:1	2:1
|	|	|	|	-	-	|	|	5:1	4:1
|	|	|	|	-	|	-	-	2:1	1:1
|	|	|	|	-	|	-	|	5:1	4:1
|	|	|	|	-	|	|	-	3:1	2:1
|	|	|	|	-	|	|	|	4:1	3:1
|	|	|	|	|	-	-	-	1:1	
|	|	|	|	|	-	-	|	3:1	2:1
|	|	|	|	|	-	|	-	2:1	1:1
|	|	|	|	|	-	|	|	3:1	2:1
|	|	|	|	|	|	-	-	1:1	
|	|	|	|	|	|	-	|	2:1	1:1
|	|	|	|	|	|	|	-	1:1	
|	|	|	|	|	|	|	|	1:1	
=	-	|	-	|	|	-		10.5:1	9.5:1
-	=	|	-	|	|	-		10.5:1	9.5:1
=	-	-	-	|	|	|		11.5:1	10.5:1
-	=	-	-	|	|	|		11.5:1	10.5:1
-	-	=	-	|	|	|		11.5:1	10.5:1
-	-	-	=	|	|	|		11.5:1	10.5:1
-	-	-	-	|	=	|		11.5:1	10.5:1
=	-	-	-	|	-	|		12.5:1	11.5:1
-	=	-	-	|	-	|		12.5:1	11.5:1
-	-	=	-	|	-	|		12.5:1	11.5:1
-	-	-	=	|	-	|		12.5:1	11.5:1
=	-	|	|	|	-	|		12.5:1	11.5:1
-	=	|	|	|	-	|		12.5:1	11.5:1
-	-	|	|	|	=	|		12.5:1	11.5:1
=	-	-	|	-	-	|		13:1	12:1
-	=	-	|	-	-	|		13:1	12:1
-	-	=	|	-	-	|		13:1	12:1
-	-	-	|	=	-	|		13:1	12:1
-	-	-	|	-	=	|		13:1	12:1
-	-	|	=	|	|	|		13.5:1	12.5:1
-	-	-	|	|	=	|		13.5:1	12.5:1
-	-	|	|	=	|	|		14:1	13:1
-	-	-	|	=	|	|		14:1	13:1
=	-	-	|	|	-	|		14.5:1	13.5:1
-	=	-	|	|	-	|		14.5:1	13.5:1
-	-	=	|	|	-	|		14.5:1	13.5:1
=	-	-	|	-	|	|		15.5:1	14.5:1
-	=	-	|	-	|	|		15.5:1	14.5:1
-	-	=	|	-	|	|		15.5:1	14.5:1

[1] https://www.eevblog.com/forum/metrology/statistical-arrays/msg3137942/#msg3137942
[2] https://www.eevblog.com/forum/metrology/diy-high-resolution-multi-slope-converter/msg3392180/#msg3392180
[3] LTZ datasheet
[4] S. A. KHAN, „The bounds of the set of equivalent resistances of n equal resistors combined in series and in parallel“