Measurement Techniques, Vol. 49, No. 6, 2006

ELECTROMAGNETIC MEASUREMENTS

THE USE OF THE ALLAN TRANSFORMATION TO ANALYZE THE MAXIMUM PERFORMANCE OF VOLTAGE STANDARDS AND COMPARATORS

A. S. Katkov

UDC 621.317.72.024.089

The problems involved when using the Allan transformation to investigate precision comparators and standards of dc voltage are considered. An approach to determining the parameters is proposed, which enables the maximum performance of the measuring instruments used to be estimated.

Key words: *Allan deviation, measurement uncertainty, dc voltage.*

Dc voltage comparators and standards are important components of a circuit for transferring the dimensions of the unit of electrical voltage from the primary standard of the volt [1]. Precision voltage comparators contain instruments for measuring voltage and comparators which enable one to choose the polarity of the voltage difference being measured and to connect the voltage standards being compared. Digital nanovoltmeters are widely used for highly accurate measurements of voltage difference. The accuracy limitations when comparing voltage standards depend mainly on the sensitivity threshold of the comparator and the instability of the voltage standards employed. These limitations, when the effect of external factors is eliminated, are due to internal noise of the voltage standards and comparators.

The sources of noise are as follows:

a) thermal noise of the resistance

$$u_R = \sqrt{4k_B TR\Delta f}\,, \tag{1}$$

where k_B is Boltzmann's constant, T is the temperature of the resistor R, and Δf is the frequency band;

b) shot noise of the current flowing I

$$i_s = \sqrt{2eI\Delta f}\,, \tag{2}$$

where e is the electron charge;

c) flicker noise, which depends on the quality of the components used in the instruments.

Thermal and shot noise have a uniform frequency spectrum, which is called white noise. For flicker noise ($1/f$ type noise), a characteristic feature is that the noise power is inversely proportional to the frequency, and hence it usually manifests itself at low frequencies.

A determination of the noise components in the measured voltage enables one to estimate adequately the applicability of statistical processing to find the result of a measurement. If the measured voltage contains mainly white noise in a

---

Translated from Izmeritel'naya Tekhnika, No. 6, pp. 49–52, June, 2006. Original article submitted February 15, 2006.

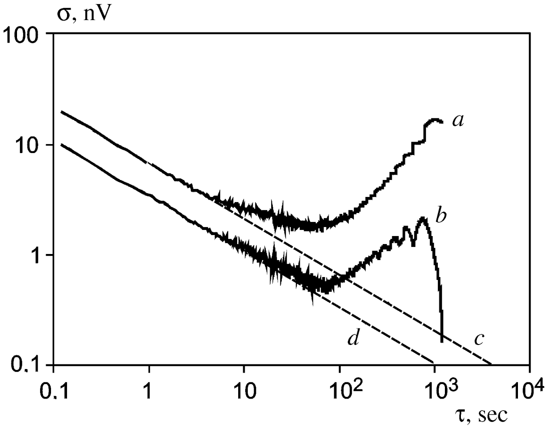

Fig. 1. Allan deviations for the comparator voltage for a resistance at the input of 1 kΩ (*a*), a short-circuited input (*b*), and voltage for thermal noise in resistances of 5.6 kΩ (*c*) and 1.4 kΩ (*d*).

specified frequency band (or in a specified measurement range), the standard uncertainty of the result of a measurement (the mean value during the measurement time) is expressed by the formula

$$\sigma_r = \sigma_m/\sqrt{N}, \tag{3}$$

where σ_m is the standard uncertainty of a series of measurements, and N is the number of measurements.

The predominant action of flicker noise excludes the use of formula (3), recommended in the manual [2], and in this case the estimate of the standard uncertainty of the result of a measurement is the standard uncertainty of a number of measurements, i.e.,

$$\sigma_r \le \sigma_m.$$

The essential difference in estimates of uncertainty of the result of a measurement as a function of the composition of the noise in the measured voltage requires an investigation of the specific types of instruments and of the choice of the method of investigation.

The spectral density of a signal is usually investigated using a Fourier transformation. This representation of a signal is useful for revealing the harmonic components, but the separation of white and flicker noise in this method may be masked by the influence of signal drift. The use of the Allan transformation [3] transfers the information on the signal parameters from the frequency domain to the time domain, which enables information to be obtained on the variable component of the voltage for a specified averaging time. This information enables one to recognize clearly the nature of both noise and deterministic changes in the measured voltage, which predominate in a specified interval of the averaging time, which, in turn, enables one to obtain the maximum sensitivity threshold of comparators, the optimum output-voltage measurement time when calibrating standards, and the also the achieved level of instability of the voltage standards being investigated.

A knowledge of the maximum performances of precision voltage comparators enables one to estimate correctly the contribution of the comparator to the list of uncertainties when comparing standards based on the Josephson effect [4]. When investigating a comparator, a voltage $u(t)$ is measured in a time interval t_0. The Allan deviation is calculated from the formula [5]

$$\sigma(\tau) = \sqrt{\sum_{n=1}^{P} [\bar{u}_{n+1}(\tau) - \bar{u}_n(\tau)]^2 / 2P},$$

TABLE 1. Parameters of Relations *a* and *b* (see Fig. 1)

Coefficient	Values of the coefficients for a resistance at the comparator input	
	input short-circuited	1 kΩ
k_0, nV/h	18	70
k_1, nV·Hz$^{-1/2}$	4.8	9.6
k_2, nV	0.2	1.2

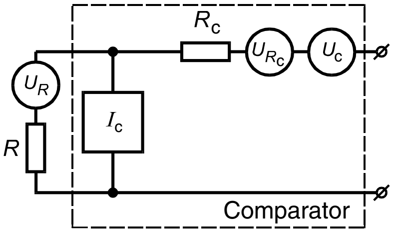

Fig. 2. Equivalent circuit of the comparator.

where $\tau = kt_0$ is the averaging interval, $P = \mathrm{Int}(M/k) - 1$ is the number of pairs of measurements, and M is the number of measurements. To analyze the voltage at the input of the comparator, a model is proposed which describes the input signal in the form of a superposition of voltages

$$F(t) = U_0 + \sum U_i \cos(\omega_i t + \varphi_i) + k_1\delta(t) + k_2\Psi(t), \tag{4}$$

where U_0 is the constant component, $\sum U_i \cos(\omega_i t + \varphi_i)$ is a set of harmonic signals, $\delta(t)$ is the model of white noise, and $\Psi(t)$ is the model of flicker noise.

If the measurement time $T \ll \pi/(2\omega)$, Eq. (4) can be written as

$$F(t) = U_0 + k_0 t + \sum U_i \cos(\omega_i t + \varphi_i) + k_1\delta(t) + k_2\Psi(t), \tag{5}$$

where $k_0 t$ is a linear approximation of the low-frequency component with a large variation period–drift.

If the harmonic component can be neglected, Eq.(5) simplifies to the following:

$$F(t) = U_0 + k_0 t + k_1\delta(t) + k_2\Psi(t). \tag{6}$$

On the basis of the above relations, the Allan deviation for the proposed model of the (6) signal has the form

$$\sigma(\tau) = \left[\frac{k_0^2\tau^2}{2} + \frac{k_1^2}{2\tau} + 2\ln 2 k_2^2\right]^{1/2}. \tag{7}$$

The conversion $\sigma(\tau)$ for U_0 is equal to zero [6].

In Fig. 1, we show the results of measurements of a comparator based on a digital nanovoltmeter in the form of the Allan deviation for a comparator with a resistance at the input of 1 kΩ (*a*) and a short-circuited input (*b*). On the graph, one

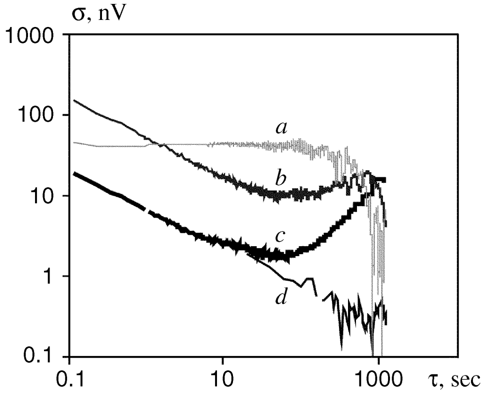

Fig. 3. Allan deviations for an output voltage of 1 V of two types of voltage standards (*a*, *b*), of a comparator without a change in polarity (*c*), and with a change in polarity (*d*) for $\tau = 10$ sec.

can distinguish regions of white noise, flicker noise, and drift. An approximation of the data obtained based on Eq. (7) gives the values of the parameters shown in Table 1.

Analysis of the parameters shows that, when the input is short-circuited the comparator has a noise k_1, equivalent to the thermal noise of a resistance of 1.4 kΩ, while for a comparator with an outer resistance of 1 kΩ – equivalent to an increase in the resistance by 5 kΩ. The increase in the noise in this case can be explained by the action of the noise current of the comparator, which flows through the input-circuit resistance [6]. A model of the equivalent circuit of the comparator is shown in Fig. 2. The noise current I_c is represented by an expression similar to (6) with coefficients $k_{0t} = 52$ pA/h, $k_{1t} = 7.3$ pA·Hz$^{-1/2}$, and $k_{2t} = 1.3$ pA.

Taking into account the action of the comparator current, the overall expression for the comparator voltage, describing the experimental curves, can be written in the form

$$\sigma(\tau) = \left[\frac{(k_0(0) + k_{0t}R)^2}{2} + \frac{k_1^2(0) + k_{1t}^2 R^2}{2\tau} + 2\ln 2(k_2^2(0) + k_{2t}^2 R^2) \right]^{1/2}. \tag{8}$$

The use of Eq. (8) enables one to obtain an averaging time τ at which the action of the white noise is completed (the second fraction in (8)) and at which flicker noise begins to predominate (the third term in (8)). For this example, the optimum duration of the measurements with the comparator $\tau = 5–20$ sec with a resistance at the comparator input of 1 kΩ. If the drift voltage of the comparator is systematically eliminated when making measurements, the value of the flicker noise can serve as an estimate of the maximum capabilities of the comparator. Some investigators have used this estimate to compare standards of the volt using the Josephson effect [7], although in this case the comparator is used to compare voltages at two polarities. As an analysis shows, switching of the connection polarity of the comparator, which is measuring in a time τ, with subsequent averaging of the results obtained leads to a reduction in the effect of the drift of the measurement circuit, and moreover, suppresses the low-frequency part of the flicker noise of the comparator to the level of white noise in accordance with the relation

$$\frac{1}{2\tau}\left(\int_{x}^{x+\tau} \sin \omega t dt - \int_{x+\tau}^{x+2\tau} \sin \omega t dt \right) \approx \frac{\tau\omega}{2} \cos \omega x \le \frac{\tau\omega}{2},$$

where $\omega \ll 1/2\tau$ defines the region of the flicker-noise spectrum.

In Fig. 3, we show graphs of the noise voltage of the comparator for bipolar measurements with an averaging time $\tau = 10$ sec (curve *d*), which shows that it is possible to reduce the sensitivity threshold of the comparator to tenths of a nanovolt when there is flicker noise and drift.

TABLE 2. Parameters of the Voltage Standards

Coefficient	Values of the coefficients for the voltage standards	
	a	*b*
k_1, nV·Hz$^{-1/2}$	14	78
k_2, nV	34	5

An investigation of the maximum parameters of the comparator may be made difficult by filtering of the noise, the action of pickup for a short averaging time and by the effect of temperature variations of the external surroundings [6] during prolonged measurements.

According to the checking scheme [8], the standards of the unit of voltage contain stabilitrons for transmitting the dimensions of the unit. The metrological parameters of the voltage standards are determined by the instability of their output voltage over time. Precision voltage standards based on stabilitrons are investigated using standards based on the Josephson effect, by comparing the output voltage of the standard and the measures using a comparator. The noise in the output voltage of the standard based on the Josephson effect can be neglected, since it amounts to less than 10^{-11} in the measurement time.

As in a comparator, the value of the flicker noise in the output voltage of a standard represents its maximum capabilities, and a comparison of the values of the white and flicker noise gives the optimum duration for carrying out measurements of the output voltage.

In Fig. 3, we show the Allan deviations for an output voltage of 1.018 V of two types of standards, and also for a voltage comparator. The results of the measurements show that for standard *a* the level of flicker noise is decisive at $\tau = 0.1$ sec, while for standard *b* it is decisive for $\tau \approx 80$ sec.

The parameters of the standards which approximate the results of the measurements in accordance with (5) are shown in Table 2.

It is well known that the output voltage of 1.018 V is generated by dividing a voltage of 10 V using a resistive divider, having an output resistance of 1 kΩ. The thermal noise of the thermal noise in a voltage in a resistance of 1 kΩ, according to (1), amounts to 4 nV·Hz$^{-1/2}$. Taking into account the fact that a current of 1 mA flows through the divider, the shot noise of which, by (2), is equal to 13 pA·Hz$^{-1/2}$, we can conclude that the shot noise current is added to the thermal noise of the resistance. This finally gives a noise of about 14 nV·Hz$^{-1/2}$, which is identical with the estimate of the parameter k_1 for voltage standard *a*. The considerable excess of the parameter k_1 for voltage standard *b* indicates the presence of additional sources of white noise. A comparison of the parameters k_2 for these standards indicates the high quality of the components (stabilitrons), employed when constructing standard *b*, which enables us to recommend measurements with it for a higher level of accuracy.

It follows from the above that the Allan transformation can be effectively used to determine the maximum capabilities of dc comparators and voltage standards. Analytical expressions have been obtained which enable one to compare comparators and voltage standards on the basis of parameters which estimate the influence of white and flicker noise. We have shown that the value of the flicker noise characterizes the maximum capabilities of the measuring instruments employed. Moreover, a comparative estimate of the parameters enables us to determine the optimum measurement time using this measuring instrument, and also the quality of the measurements for a corresponding reduction in the uncertainty in the result of the measurement.

REFERENCES

1. V. S. Aleksandrov, A. S. Katkov, and G. P. Telitchenko, *Izmer. Tekh.*, No. 3, 6 (2002).
2. *Guide to the Expression of Uncertainty in Measurement*, ISO, Geneva (1995).

3. D. W. Allan, *IEEE Trans. Instrum. Meas.*, **IM-36**, No. 2, 646 (1987).
4. R. Behr, J. Niemeyer, and A. Katkov, *IEEE Trans. Instrum. Meas.*, **50,** No. 2, 203 (2001).
5. T. J. Witt and D. Reymann, *IEEE Proc. Sci. Meas. Tech.*, **147,** No. 4, 177 (2000).
6. T. J. Witt, *IEEE Trans. Instrum. Meas.*, **50,** No. 2, 445 (2001).
7. R. Behr and A. S. Katkov, *Metrologia*, **42**, No. 1A, Tech. Suppl. 01005 (2005).
8. GOST 8.027-2001. GSI, *The State Checking Scheme for Instruments for Measuring DC Voltage and Electromotive Force.*